



# **Math Days in SICM**

Lie Vertex Workshop

Cuipo Jiang
Kazuya Kawasetsu
João Fernado Schwarz
Xuanzhong Dai
Olivier Mathieu

Genqiang Liu
Husileng Xiao
Ana Kontrec
Hao Li
Xingpeng Liu
Zhenyu Zhou





# **Schedule**





### Nov. 19 2025

10:00 - 10:50 Cuipo Jiang

**11:00 - 11:50** Kazuya Kawasetsu

12:10 - 14:00 Break

**14:30 - 15:10** João Fernado Schwarz

**15:20 - 16:00** Xuanzhong Dai

16:20 - 17:10 Olivier Mathieu

### Nov. 20 2025

**10:00 - 10:50** Genqiang Liu

**11:00 - 11:50** Husileng Xiao

12:10 - 14:00 Break

14:00 - 14:40 Ana Kontrec

14:45 - 15:25 Hao Li

**15:35 - 16:15** Xingpeng Liu

16:20 - 17:00 Zhenyu Zhou





# On singular vectors of vacuum modules

Cuipo Jiang

Nov. 19 2025, 10:00 - 10:50

### **Abstract**

I will talk about some recent progress on determining wrights of singular vectors of universal affine vertex operator algebras.





# Representation theory of affine vertex operator algebras

Kazuya Kawasetsu

Nov. 19 2025, 11:00 - 11:50

### **Abstract**

In the study of vertex operator algebras, those related to rational 2d conformal field theory (rational vertex operator algebras) have been investigated extensively. Recently, irrational ones attract much more attention, partly due to their appearance in 4d conformal field theory via 4d/2d duality.

In this talk, we will explain some recent developments in representation theory of irrational affine vertex operator algebras, which have affine Lie algebra symmetry.





### Harish-Chandra modules revisited

João Fernado Schwarz

Nov. 19 2025, 14:30 - 15:10

### **Abstract**

The theory of Harish-Chandra modules is very important in Lie theory. For generalized Weyl algebras, a related notion of generalized weight modules is also central. We work in an abstract framework introduced in 1994 by Yu. Drozd, V. Futorny and S. Ovsienko, which unifies many variations of the notion of a Harish-Chandra module, such as those mentioned above. An important role is played by a suitable big subalgebra, called Harish-Chandra subalgebra. We show some ring theoretical results about them, study many tools to compare Harish-Chandra categories, and show applications to rational Cherednik algebras and spherical Coulomb branch algebras





# Quasi-lisse vertex algebras appearing from BRST reduction

Xuanzhong Dai

Nov. 19 2025, 15:20 - 16:00

### **Abstract**

In this talk, we present a uniform geometric framework that connects the representation theory of vertex algebras with symplectic geometry and invariant theory. We construct chiral analogues of differential operators acting on classical invariant rings, realized as global sections of sheaves of chiral differential operators associated with vector bundles on smooth open subvarieties of affine GIT quotients, using the BRST reduction. As an application, we construct new infinite families of simple conformal quasi-lisse vertex algebras. This is joint work with Tomoyuki Arakawa and Bailin Song.



# On the classification of simple $\mathbb{Z}^n$ -graded Lie algebras. In memory of Kenji Iohara, 1971-2025

Olivier Mathieu

Nov. 19 2025, 16:20 - 17:10

### **Abstract**

Given an algebra A, a field K containing all structure constants relative to some basis is called a *field of definition* of the algebra A.

We are interested in certain classes  $\mathcal C$  of infinite dimensional algebras. The class  $\mathcal C$  is called of *finite type* if all algebras  $A \in \mathcal C$  are defined over a number field. Otherwise  $\mathcal C$  is called *tame* if all  $A \in \mathcal C$  are defined over fields of transcendental dimension  $\leq 1$ . The class  $\mathcal C$  is called *wild* if all algebras are  $A \in \mathcal C$  are defined over fields of finite, but unbounded, transcendental dimension. A *monster* is an algebra  $A \in \mathcal C$  which cannot be defined over a field of finite transcendental dimension. A class  $\mathcal C$  containing a monster is called *superwild*.

The class of simple finitely generated  $\mathbb{Z}$ -graded is wild and it is a numerable union of algebraic varieties. Moreover we had classified those of finite growth: they are all defined over a number field.

By contrast, I conjecture that for  $n \geq 2$ , the class of simple finitely generated  $\mathbb{Z}^n$ -graded Lie algebras is super-wild. However, in two long papers with Kenji Iohara, we have classified all simple finitely generated  $\mathbb{Z}^n$ -graded Lie algebras whose all homogenous components have dimension 1. Beside two affine Lie algebras, there is a n-parameter family of Witt-Yu-algebras.

However this class is not an union of algebraic varieties.





# Irreducible cuspidal $\mathfrak{sl}_{n+1}$ -modules from finite-dimensional modules over the minimal nilpotent finite W-algebra

Genqiang Liu

Nov. 20 2025, 10:00 - 10:50

### **Abstract**

A weight  $\mathfrak{gl}_{n+1}$ -module with finite-dimensional weight spaces is called a cuspidal module,if every root vector of  $\mathfrak{gl}_{n+1}$  acts injectively on it.

We have shown that any block with a generalized central character of the cuspidal  $\mathfrak{gl}_{n+1}$ -module category is equivalent to a block of the category of finite-dimensional modules over the minimal nilpotent finite W-algebra W(e) for  $\mathfrak{gl}_{n+1}$ . In this talk, using a centralizer realization of W(e) and an explicit embedding  $W(e) \to U(\mathfrak{gl}_n)$ , we show that every finite-dimensional irreducible W(e)-module is isomorphic to an irreducible W(e)-quotient module of some finite-dimensional irreducible  $\mathfrak{gl}_n$ -module  $V(\lambda)$ . We also completely determine the irreducibility of arbitrary  $V(\lambda)$  when it was viewed as a W(e)-module. As an application, we can give very explicit realizations of all irreducible cuspidal  $\mathfrak{gl}_{n+1}$ -modules using finite-dimensional irreducible  $\mathfrak{gl}_n$ -modules, without using the twisted localization method and the coherent family introduced by Mathieu.





# An introduction to super nilpotent orbits

Husileng Xiao

Nov. 20 2025, 11:00 - 11:50

### **Abstract**

The geometry of nilpotent orbits in a Lie algebra  $\mathfrak{g}_0$  is fundamental to its representation theory. We aim to develop an analogous theory for Lie superalgebras. Musson introduced the nilpotent orbits of Lie superalgebra in 2005. But their relations with the representation theory of  $\mathfrak{g}_0$  is still open. In this talk we prove that they arise as the associated varieties of simple modules of. We also explore their connection with the BeilinsonBernstein localization of  $\mathfrak{g}$ -modules with atypical weights.





# Kazama-Suzuki duality between certain simple Walgebras

Ana Kontrec

Nov. 20 2025, 14:00 - 14:40

### **Abstract**

One of the most important families of vertex algebras are affine vertex algebras and their associated  $\mathcal{W}$ -algebras, which are connected to various aspects of geometry and physics. The notion of Kazama-Suzuki dual was first introduced in the context of the duality of the N=2 superconformal algebra and affine Lie algebra  $\mathfrak{sl}(2)$ .

I will present some old and new Kazama-Suzuki dualities between affine W-algebras and vertex superalgebras.

This is joint work with D. Adamovic.





# Tensor Categories of W-Algebras

Hao Li

Nov. 20 2025, 14:45 - 15:25

### **Abstract**

I will discuss tensor categories arising from W-algebras, beginning with Arakawa's results on rational principal W-algebras and their tensor categories. For nonrational W-algebras in rank one, namely Virasoro vertex operator algebras, I will review recent progress on Virasoro tensor categories by McRae, Yang, and others, and then on the progress on Virasoro tensor categories at positive rational central charge (joint with McRae and Yang). If time permits, I will also indicate related developments for higher-rank W-algebras (joint with Arakawa).





Xingpeng Liu

Nov. 20 2025, 15:35 - 16:15

modules over quantum groups

### **Abstract**

In this talk, I will discuss the growth behavior of the dimensions of irreducible representations for De Concini–Kac quantum groups as the order of the root of unity varies. These representations exhibit polynomial growth in the order of the root of unity, and the degree of this polynomial coincides with the Gelfand–Kirillov dimension of the corresponding irreducible highest weight module at generic q. We then determine the minimal possible nonzero growth and explicitly construct the highest weight representations with this property for types A, B, and C. As an application, we prove that quantum cuspidal modules (at generic q) can occur only when the underlying semisimple Lie algebra has simple components of type A, B, or C, thereby revealing a distinction from the classical case.





# Full conformal oscillator representations of orthogonal Lie algebras and combinatorial identities

Zhenyu Zhou

Nov. 20 2025, 16:20 - 17:00

### **Abstract**

Zhao and Xu (2013) constructed a functor from the category of  $\mathfrak{o}(k)$ -modules to the category of  $\mathfrak{o}(k+2)$ -modules. We use this functor successively to obtain an inhomogeneous first-order differential operator realization for any highest-weight representation of  $\mathfrak{o}(2n+3)$  in  $(n+1)^2$  variables and that of  $\mathfrak{o}(2n+2)$  in n(n+1) variables. When the highest weight is dominant integral, we determine the corresponding finite-dimensional irreducible module explicitly. We also find an equation of counting the dimension of a finite-dimensional irreducible  $\mathfrak{o}(k+2)$ -modules in terms of certain alternating sum of the dimensions of irreducible  $\mathfrak{o}(k)$ -modules. In the case of the Steinberg modules, we obtain new sum-product combinatorial identities of finite type.

# **Conference Information**







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