

# Towards a VEM formulation for hyperbolic problems

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We show, in [1] how to combine in a natural way (i.e. without any test nor switch) the conservative and non conservative formulations of an hyperbolic system that has a conservative form. This is inspired from two different class of schemes: the Residual Distribution one [2], and the Active Flux formulations [3, 4, 5, 6, 7]. The solution is globally continuous, and as in the active flux method, described by a combination of point values and average values. Unlike the "classical" active flux methods, the meaning of the point-wise and cell average degrees of freedom is different, and hence follow different form of PDEs: it is a conservative version of the cell average, and a possibly non conservative one for the points. This new class of scheme is proved to satisfy a Lax-Wendroff like theorem. We also develop a method to perform non linear stability. We illustrate the behaviour on several benchmarks, some quite challenging.

An extension to 2D with triangular mesh is provided, with several example showing that smooth and non smooth problems can be handled efficiently.

## References

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