High-order Structure-Preserving Schemes for Special Relativistic Hydrodynamics

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ABSTRACT
This talk mainly reviews two high-order accurate structure-preserving finite difference schemes for the special relativistic hydrodynamics (RHD). The first is the physical-constraints-preserving (PCP) scheme, which preserves the positivity of the rest-mass density and the pressure and the bounds of the fluid velocity and is built on the local Lax-Friedrichs (LxF) splitting, the WENO reconstruction, the PCP flux limiter, and the high-order strong stability preserving time discretization. The key to developing such scheme is to prove the convexity and other properties of the admissible state set and to discover a concave function with respect to the conservative vector. The second is the entropy stable (ES) scheme, whose semi-discrete version satisfies the entropy inequality. The key is to technically construct the affordable entropy conservative (EC) flux of the semi-discrete second-order accurate EC schemes satisfying the semi-discrete entropy equality for the found convex entropy pair. As soon as the EC flux is derived, the dissipation term can be added to give the semi-discrete ES schemes satisfying the semi-discrete entropy inequality. The WENO reconstruction for the scaled entropy variables and the previous time discretization are implemented to obtain the fully-discrete high-order "ES" schemes. The performance of the proposed schemes has been demonstrated by numerical experiments. By the way, we also briefly review other relative works on the structure-preserving schemes for the special RHDs. Those works have been further to the general equation of state and the special relativistic magnetohydrodynamics etc., see our papers listed below for details.

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