Multi-dimensionally aware Entropy stable and positivity preserving Godunov-type schemes for hydrodynamics on unstructured grid

Raphaël Loubère[a]

[a]Joint Research Laboratory "Anabase" and Institut de Mathmatiques de Bordeaux (IMB), Univ. Bordeaux, CNRS, Bordeaux INP, UMR 5251, F-33400 Talence, (France)

raphael.loubere@u-bordeaux.fr

This joint work has been made with the authors of [1, 2, 3, 4].

In this paper, we propose to reuse the notion of simple Riemann solver in Lagrangian coordinates following Gallice [6] to develop a new Eulerian Finite Volume (FV) scheme in the multi-dimensional case on unstructured meshes [1, 2, 3, 4]. In [2], as a proof of concept we entirely derive the associated first-order accurate cell-centered Eulerian scheme for compressible flows using the Lagrangian to Eulerian correspondence. First, the Lagrangian simple Riemann solver [6] is used as a building block to construct its Eulerian counterpart. This solver inherits by construction the properties of the Lagrangian one, mainly: positivity preservation, entropy dissipation, well-defined CFL condition and wave-speed ordering. From this Riemann solver, a classical two-point first-order Finite Volume Eulerian scheme can be deduced for which the numerical fluxes of a given cell are computed only with respect to two neighbors through a common face. Next, we introduce another Eulerian numerical scheme which involves a multi-dimensional Lagrangian nodal solver [5] leading to the so-called multi-point Riemann solver that involves all surrounding cells, including corner cells. The conservation is no more relying on a one-to-one flux cancellation across a face like for most FV approach. Conversely, in this work conservation is retrieved on a node basis. An associated first-order Eulerian scheme is derived on the basis of this multi-point nodal-based Riemann solver. We prove that this FV multi-point scheme still inherits some good properties (entropic, positive) with the extra-property of coupling all neighbor cells in a consistent way. A set of numerical results on general 2D unstructured grids are presented on several classical two-dimensional test cases, showing that the two-point scheme generates spurious instabilities such as the infamous carbuncle phenomena, while the multi-point scheme seems insusceptible to those [8, 2]. Classical and demanding test case results such as Sedov, Noh, Forward Facing step, vortex, odd-even decoupling, etc. will be presented and commented.

If time permits some extensions of this family of Eulerian FV numerical schemes will also be presented, namely the second-order in space/time extension, the 3D and parallel version of these schemes, especially on hypersonic reentry test cases. At last the treatment of balanced laws has also been studied by solving the Shallow-Water toy model in 2D, [3]. Numerical tests for such extensions will be systematically presented to illustrate the good behavior of this family of Eulerian FV schemes.

References

[1] A. Chan, G. Gallice, R. Loubre, and P.-H. Maire. Positivity preserving and entropy consistent approximate Riemann solvers dedicated to the high-order MOOD-based Finite Volume discretization of Lagrangian and Eulerian gas dynamics. *Computers and Fluids*, 229, 105056, /10.1016/j.compfluid.2021.105056. 2021.

[2] G. Gallice, A. Chan, R. Loubre, and P.-H. Maire. Entropy stable and positivity preserving Godunov-type schemes for multidimensional hyperbolic systems on unstructured grid *Journal of Computational Physics*, 468, 2022

[3] Alessia Del Grosso, M. J. Castro, A. Chan, G. Gallice, R. Loubère, P.-H. Maire A well-balanced and positive finite volume scheme for shallow-water equations in multi-dimensions for unstructured grids derived from Lagrangian roots. *Journal of Computational Physics*, submitted 2023

[4] V. Delmas, R. Loubère, and P.-H. Maire. A subface based multi-dimensionally aware finite volume scheme for diffusion on unstructured and irregular 3D grids in preparation, 2023

[5] P.-H. Maire A high-order cell-centered Lagrangian scheme for two-dimensional compressible fluid flows on unstructured meshes. *Journal of Computational Physics*, 229(12):4724?4761, 2010.

[6] G. Gallice. Positive and Entropy Stable Godunov-Type Schemes for Gas Dynamics and MHD Equations in Lagrangian or Eulerian Coordinates. *Numer. Math.*, 94(4):673?713, 2003.

[7] Z. Shen, W. Yan, and G. Yuan A robust and contact resolving Riemann solver on unstructured mesh, Part I, Euler method *Journal of Computational Physics*, 268:432?455, 2014.

[8] A.V. Rodionov. Artificial viscosity Godunov-type schemes to cure the carbuncle phenomenon. *Journal of Computational Physics*, 345:308?329, 2017.