```
> with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(1)**

```
> interface(rtablesize=infinity);
```

$$\infty \quad (2)$$

```
> MltplMtr := Matrix(11, 11, [
> [0, h[10], h[1], e[10], e[1], e[11], e[21], f[21], f
  [11], f[1], f[10]],
> [h[10], 0, 0, 2*e[10], -2*e[1], 0, 2*e[21], -2*f[21],
  0, 2*f[1], -2*f[10]],
> [h[1], 0, 0, -e[10], 2*e[1], e[11], 0, 0, -f[11], -2*f
```

```
  [1], f[10]],
> [e[10], -2*e[10], e[10], 0, e[11], e[21], 0, -2*f[11],
  -2*f[1], 0, h[10]],
> [e[1], 2*e[1], -2*e[1], -e[11], 0, 0, 0, 0, f[10], h
  [1], 0],
> [e[11], 0, -e[11], -e[21], 0, 0, 0, 2*f[10], h[10] + 2*
  h[1], e[10], -2*e[1]],
> [e[21], -2*e[21], 0, 0, 0, 0, 0, 4*h[10] + 4*h[1], 2*e
  [10], 0, -2*e[11]],
> [f[21], 2*f[21], 0, 2*f[11], 0, -2*f[10], -4*h[10] - 4*
  h[1], 0, 0, 0, 0],
> [f[11], 0, f[11], 2*f[1], -f[10], -h[10] - 2*h[1], -2*e
  [10], 0, 0, 0, f[21]],
> [f[1], -2*f[1], 2*f[1], 0, -h[1], -e[10], 0, 0, 0, 0, f
  [11]],
> [f[10], 2*f[10], -f[10], -h[10], 0, 2*e[1], 2*e[11], 0,
  -f[21], -f[11], 0]]);
```

MltplMtr := **(3)**

	1	2	3	4	5	6	7	
1	0	h_{10}	h_1	e_{10}	e_1	e_{11}	e_2	...
2	h_{10}	0	0	$2\ e_{10}$	$-2\ e_1$	0	$2\ \epsilon$	...
3	h_1	0	0	$-e_{10}$	$2\ e_1$	e_{11}	0	...
4	e_{10}	$-2\ e_{10}$	e_{10}	0	e_{11}	e_{21}	0	...
5	e_1	$2\ e_1$	$-2\ e_1$	$-e_{11}$	0	0	0	...
6	e_{11}	0	$-e_{11}$	$-e_{21}$	0	0	0	...
7	e_{21}	$-2\ e_{21}$	0	0	0	0	0	...
8	f_{21}	$2\ f_{21}$	0	$2\ f_{11}$	0	$-2\ f_{10}$	$-4\ h_{10}$	...
9	f_{11}	0	f_{11}	$2\ f_1$	$-f_{10}$	$-h_{10}-2\ h_1$	-2	...
10	f_1	$-2\ f_1$	$2\ f_1$	0	$-h_1$	$-e_{10}$	0	...
11	f_{10}	$2\ f_{10}$	$-f_{10}$	$-h_{10}$	0	$2\ e_1$	$2\ \epsilon$	...

```
> # List perfect monomials, see formulas 6.4:
> c[1] = (f[1]) . (e[1]);
  c[2] = (f[21]) . (e[21]);
  c[3] = (f[10]) . (e[10]);
  c[4] = (f[11]) . (e[11]);
  c[5] = (f[11]) . (e[1]) . (e[10]);
  c[6] = (f[10]) . (f[1]) . (e[11]);
  c[7] = (f[21]) . (e[11]) . (e[10]);
  c[8] = (f[10]) . (f[11]) . (e[21]);

  c[9] = (f[21]) . (e[1]) . (e[10]) . (e[10]);

  c[10] = (f[10]). (f[10]) . (f[1]) . (e[21]);

  c[11] = (f[1]) . (f[21]) . (e[11]) . (e[11]);

  c[12] = (f[11]) . (f[11])  . (e[21]) . (e[1]);
```

$$c_1 = f_1 \bullet e_1$$

$$c_2 = f_{21} \bullet e_{21}$$

$$c_3 = f_{10} \bullet e_{10}$$

$$c_4 = f_{11} \bullet e_{11}$$

$$c_5 = f_{11} \bullet e_1 \bullet e_{10}$$

$$c_6 = f_{10} \bullet f_1 \bullet e_{11}$$

$$c_7 = f_{21} \bullet e_{11} \bullet e_{10}$$

$$c_8 = f_{10} \bullet f_{11} \bullet e_{21}$$

$$c_9 = f_{21} \bullet e_1 \bullet e_{10}^2$$

$$c_{10} = f_{10}^2 \bullet f_1 \bullet e_{21}$$

$$c_{11} = f_1 \bullet f_{21} \bullet e_{11}^2$$

$$c_{12} = f_{11}^2 \bullet e_{21} \bullet e_1 \quad (4)$$

------ Start ---------

```
> with(LinearAlgebra);
> interface(rtablesize=infinity);
```

$$\infty \tag{1.1}$$

```
> UM := proc( TT::Matrix ) ::Matrix;
> local   i,j,k,r,n,Mm,Mt,xx;
>    Mm := Matrix(TT);
>    Mt := Matrix( 6,6,0);
>    for i from 1 to 6 do
>      for j from 1 to 6 do
>        xx := factor(Mm[i,j]);
>        Mt[i,j] := collect( xx, SSSS, distributed,
  factor);
>      od;
>    od;
>    return Mt;
>  end;
```

(1.2)

```
UM := proc(TT::Matrix)::Matrix;
   local i, j, k, r, n, Mm, Mt, xx;
   Mm := Matrix(TT);
   Mt := Matrix(6, 6, 0);
   for i to 6 do
      for j to 6 do
         xx := factor(Mm[i, j]);
         Mt[i, j] := collect(xx, SSSS, distributed, factor)
      end do
   end do;
   return Mt
end proc
```

```
> UM4 := proc( TT::Matrix ) ::Matrix;
> local   i,j,k,r,n,Mm,Mt,xx;
>    Mm := Matrix(TT);
>    Mt := Matrix( 4,4,0);
>    for i from 1 to 4 do
>      for j from 1 to 4 do
>        xx := factor(Mm[i+1,j+1]);
>        Mt[i,j] := collect( xx, SSSS, distributed,
  factor);
```

```
>       od;
>     od;
>     return Mt;
>   end;
```

UM4 := proc(TT::Matrix)::Matrix; **(1.3)**

```
UM4 := proc(TT::Matrix)::Matrix;
   local i, j, k, r, n, Mm, Mt, xx;
   Mm := Matrix(TT);
   Mt := Matrix(4, 4, 0);
   for i to 4 do
      for j to 4 do
         xx := factor(Mm[i + 1, j + 1]);
         Mt[i, j] := collect(xx, SSSS, distributed, factor)
      end do
   end do;
   return Mt
end proc
```

```
> UU := proc( T )
>     local   U;
>     U := collect( T, SSSS, distributed, factor);
>     return U;
> end;
```

(1.4)

```
UU := proc(T)
   local U;
   U := collect(T, SSSS, distributed, factor); return U
end proc
```

```
> SSSS := {Id};
```

$$SSSS := \{Id\}$$ **(1.5)**

```
# List all relations used for calculations:
> #h[3] := h[10] + h[1];
> h[10] := h[3] - h[1];
> # List Relations
> Rc5c1 := c[6]+c[5].c[1]+c[5]*(-h[1]-1)-c[4]*h[1]-c[1]
```

```
.c[5]-c[1].c[4]+c[1].c[3];

> Rc7c2 := c[7].c[2]+c[7]*(4*h[1]+4*h[10])-c[2].c[7]+2*
  (c[2].c[4])-2*(c[2].c[3])+c[2]*(4*h[1]+2*h[10]);

> Rc5c3 := -c[9]-c[8]+2*c[6]+c[5].c[3]-c[5]*h[10]-c[3].
  c[5]+c[3].c[4]-2*(c[1].c[3]);

> Rc7c3 := 2*c[9]+2*c[8]+c[7].c[3]-c[7]*h[10]-4*c[6]-c
  [3].c[7]-2*(c[3].c[4])+c[2].c[3];

> Rc6c5 := c[11].c[3]+6*c[11]+c[9].c[3]+c[9]*(-2*h[10]
  -6)-6*c[8]+6*c[7]+20*c[6]+4*c[5]-c[4].c[7]-2*(c[4].c
  [6])-4*c[4]*h[1]-c[3].c[11]-c[3].c[9]+c[3].c[7]-2*(c
  [3].c[5])+8*(c[3].c[4])+2*(c[2].c[6])-c[2].c[3]+6*c
  [2]+4*(c[1].c[7])-4*(c[1].c[4])-4*(c[1].c[3]);

> Rc5c6 := -c[10]-2*c[8]+c[7]+c[6].c[5]+c[6]*(-h[10]+2)
  +c[4].c[5]+c[3].c[6]+2*(c[3].c[4])+c[1].c[8]-2*(c[1].
  c[6])-c[1].c[3].c[4]+Typesetting[delayDotProduct](c
  [1].c[3], -2*h[1]-h[10]-2, true);
```

$$h_{10} := h_3 - h_1$$

$$Rc5c1 := c_6 + c_5 \cdot c_1 + c_5 \left(-h_1 - 1\right) - c_4\ h_1 - c_1 \cdot c_5 - c_1 \cdot c_4 + c_1 \cdot c_3$$

$$Rc7c2 := c_7 \cdot c_2 + 4\ c_7\ h_3 - c_2 \cdot c_7 + 2\ c_2 \cdot c_4 - 2\ c_2 \cdot c_3 + c_2 \left(2\ h_1 + 2\ h_3\right)$$

$$Rc5c3 := -c_9 - c_8 + 2\ c_6 + c_5 \cdot c_3 - c_5 \left(h_3 - h_1\right) - c_3 \cdot c_5 + c_3 \cdot c_4 - 2\ c_1 \cdot c_3$$

$$Rc7c3 := 2\ c_9 + 2\ c_8 + c_7 \cdot c_3 - c_7 \left(h_3 - h_1\right) - 4\ c_6 - c_3 \cdot c_7 - 2\ c_3 \cdot c_4 + c_2 \cdot c_3$$

$$Rc6c5 := c_{11} \cdot c_3 + 6\ c_{11} + c_9 \cdot c_3 + c_9 \left(-2\ h_3 + 2\ h_1 - 6\right) - 6\ c_8 + 6\ c_7 + 20\ c_6 + 4\ c_5 - c_4 \cdot c_7 - 2\ c_4 \cdot c_6 - 4\ c_4\ h_1 - c_3 \cdot c_{11} - c_3 \cdot c_9 + c_3 \cdot c_7 - 2\ c_3 \cdot c_5 + 8\ c_3 \cdot c_4 + 2\ c_2 \cdot c_6 - c_2 \cdot c_3 + 6\ c_2 + 4\ c_1 \cdot c_7 - 4\ c_1 \cdot c_4 - 4\ c_1 \cdot c_3$$

$$Rc5c6 := -c_{10} - 2\ c_8 + c_7 + c_6 \cdot c_5 + c_6 \left(-h_3 + h_1 + 2\right) + c_4 \cdot c_5 + c_3 \cdot c_6 + 2\ c_3 \cdot c_4 + c_1 \cdot c_8 - 2\ c_1 \cdot c_6 - c_1 \cdot c_3 \cdot c_4 + \left(c_1 \cdot c_3\right) \left(-h_1 - h_3 - 2\right) \quad \textbf{(2.1)}$$

```
> # List Cazimirs
> Caz1 := UU(Id*(2*h[1]^2+2*(h[10]*h[1])+6*h[1]+h[10]
```

```
^2+4*h[10])+2*c[4]+2*c[3]+c[2]+4*c[1]);

> Caz2 := 2*c[12]+2*c[11]-2*c[10]-2*c[9]+c[8]*(2*h[1]
  +1)+c[7]*(2*h[1]-1)+c[6]*(4*h[1]+4*h[10]+6)+c[5]*(4*h
  [1]+4*h[10]+10)-c[4]^2+c[4]*(-2*h[1]^2-2*h[1]*h[10]
  -2*h[1]+2*h[10]+6)-2*(c[3].c[4])-c[3]^2+c[3]*(2*h[1]
  ^2+2*h[1]*h[10]+6*h[1]+2*h[10]+6)-c[2]*(h[1]-1)*(h[1]
  +1)-4*(c[1].c[2])-4*c[1]*(h[1]+h[10]+3)*(h[1]+h[10]
  +1)-h[1]*(h[1]+2)*(h[1]+h[10]+3)*(h[1]+h[10]+1)*Id;
```

$$Caz1 := 2\ c_4 + 2\ c_3 + c_2 + 4\ c_1 + \left(h_1^2 + h_3^2 + 2\ h_1 + 4\ h_3\right)\ Id$$

$$Caz2 := 2\ c_{12} + 2\ c_{11} - 2\ c_{10} - 2\ c_9 + c_8\ (2\ h_1 + 1) + c_7\ (2\ h_1 - 1) + c_6\ (4\ h_3 + 6) + c_5\ (4\ h_3 + 10) - c_4^2 + c_4\ \left(-2\ h_1^2 - 2\ h_1\ (h_3 - h_1) - 4\ h_1 + 2\ h_3 + 6\right) - 2\ c_3 \cdot c_4 - c_3^2 + c_3\ \left(2\ h_1^2 + 2\ h_1\ (h_3 - h_1) + 4\ h_1 + 2\ h_3 + 6\right) - c_2\ (h_1 - 1)\ (h_1 + 1) - 4\ c_1 \cdot c_2 - 4\ c_1\ (h_3 + 3)\ (h_3 + 1) - h_1\ (h_1 + 2)\ (h_3 + 3)\ (h_3 + 1)\ Id \quad \textbf{(2.2)}$$

```
> #  Replace perfect elements
> c[12] := -c[10]-c[8].c[1]-2*c[8]-c[2]+c[1].c[8];

> c[11] := -c[9]+c[7].c[1]-2*c[7]-c[2]-c[1].c[7];
> c[10] := c[9]-c[8].c[1]-c[8]+c[7]+(1/2)*c[5].c[2]-
  (1/2)*c[2].c[5]+c[1].c[8];
> c[9] := (1/2)*(c[7].(c[1]^2))/h[1]+(1/2)*Typesetting
  [delayDotProduct](c[7].c[1], h[1]-2, true)/h[1]-2*c
  [7]-c[2]-(c[1].c[7].c[1])/h[1]-(1/2)*Typesetting
  [delayDotProduct](c[1].c[7],h[1]+2,true)/h[1]-(c[1].c
  [2])/h[1]+(1/2)*(c[1].c[1].c[7])/h[1];
> c[8] := c[7]-(1/2)*c[3].c[2]+(1/2)*c[2].c[3];
> c[7] := -(1/16)*(c[3].(c[2]^2))/(h[1]+h[10])+(1/4)*
  Typesetting[delayDotProduct](c[3].c[2], h[1]+h[10]+2,
  true)/(h[1]+h[10])-(1/2)*(c[2].c[4])/(h[1]+h[10])+
  (1/8)*(c[2].c[3].c[2])/(h[1]+h[10])-(1/4)*c[2].c[3]-
  (1/16)*(c[2].c[2].c[3])/(h[1]+h[10])-(1/2)*c[2]*(2*h
  [1]+h[10])/(h[1]+h[10]);
> c[6] := c[5]+c[3].c[1]-c[1].c[3];
```

```
> c[5] := -c[4]-(1/2)*(c[3].(c[1]^2))/h[1]-(1/2) *
  Typesetting[delayDotProduct](c[3].c[1], h[1]-2, true)
  /h[1]-(c[1].c[4])/h[1]+(c[1].c[3].c[1])/h[1]+(1/2)*c
  [1].c[3]-(1/2)*(c[1].c[1].c[3])/h[1];
> c[4] := UU(-c[3]-(1/2)*c[2]-2*c[1]+Id*(-h[1]^2-h[10]*
  h[1]-(1/2)*h[10]^2+(1/2)*z[1]-3*h[1]-2*h[10]));
```

$$c_{12} := -c_{10} - c_8 \cdot c_1 - 2\ c_8 - c_2 + c_1 \cdot c_8$$

$$c_{11} := -c_9 + c_7 \cdot c_1 - 2\ c_7 - c_2 - c_1 \cdot c_7$$

$$c_{10} := c_9 - c_8 \cdot c_1 - c_8 + c_7 + \frac{c_5 \cdot c_2}{2} - \frac{c_2 \cdot c_5}{2} + c_1 \cdot c_8$$

$$c_9 := \frac{c_7 \cdot c_1^2}{2\ h_1} + \frac{(c_7 \cdot c_1)\ (h_1 - 2)}{2\ h_1} - 2\ c_7 - c_2 - \frac{c_1 \cdot c_7 \cdot c_1}{h_1} - \frac{(c_1 \cdot c_7)\ (h_1 + 2)}{2\ h_1} - \frac{c_1 \cdot c_2}{h_1} + \frac{c_1^2 \cdot c_7}{2\ h_1}$$

$$c_8 := c_7 - \frac{c_3 \cdot c_2}{2} + \frac{c_2 \cdot c_3}{2}$$

$$c_7 := -\frac{c_3 \cdot c_2^2}{16\ h_3} + \frac{(c_3 \cdot c_2)\ (h_3 + 2)}{4\ h_3} - \frac{c_2 \cdot c_4}{2\ h_3} + \frac{c_2 \cdot c_3 \cdot c_2}{8\ h_3} - \frac{c_2 \cdot c_3}{4} - \frac{c_2^2 \cdot c_3}{16\ h_3} - \frac{c_2\ (h_3 + h_1)}{2\ h_3}$$

$$c_6 := c_5 + c_3 \cdot c_1 - c_1 \cdot c_3$$

$$c_5 := -c_4 - \frac{c_3 \cdot c_1^2}{2\ h_1} - \frac{(c_3 \cdot c_1)\ (h_1 - 2)}{2\ h_1} - \frac{c_1 \cdot c_4}{h_1} + \frac{c_1 \cdot c_3 \cdot c_1}{h_1} + \frac{c_1 \cdot c_3}{2} - \frac{c_1^2 \cdot c_3}{2\ h_1}$$

$$c_4 := -c_3 - \frac{c_2}{2} - 2\ c_1 + \left(-\frac{1}{2}\ h_1^2 - \frac{1}{2}\ h_3^2 + \frac{1}{2}\ z_1 - h_1 - 2\ h_3\right)\ Id \quad \textbf{(2.3)}$$

Define Matrix and relations

```
> SSSS := {z[1],e[1,1],e[2,2],e[3,3],e[4,4],e[5,5],e[6,
  6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e
  [5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,3}, e_{3,4}, e_{4,3}, e_{4,4}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(3.1)}$$

```
> # We use only 6*6 Matrix from infinite matrix. ======
  ================
> c[3] := Matrix(6, 6, [
> [e[1,1],e[1,2],e[1,3],e[1,4],e[1,5],e[1,6]],
> [e[2,1],e[2,2],e[2,3],e[2,4],e[2,5],e[2,6]],
> [e[3,1],e[3,2],e[3,3],e[3,4],e[3,5],e[3,6]],
> [e[4,1],e[4,2],e[4,3],e[4,4],e[4,5],e[4,6]],
> [e[5,1],e[5,2],e[5,3],e[5,4],e[5,5],e[5,6]],
> [e[6,1],e[6,2],e[6,3],e[6,4],e[6,5],e[6,6]]]);
```

$$c_3 := \begin{bmatrix} e_{1,1} & e_{1,2} & e_{1,3} & e_{1,4} & e_{1,5} & e_{1,6} \\ e_{2,1} & e_{2,2} & e_{2,3} & e_{2,4} & e_{2,5} & e_{2,6} \\ e_{3,1} & e_{3,2} & e_{3,3} & e_{3,4} & e_{3,5} & e_{3,6} \\ e_{4,1} & e_{4,2} & e_{4,3} & e_{4,4} & e_{4,5} & e_{4,6} \\ e_{5,1} & e_{5,2} & e_{5,3} & e_{5,4} & e_{5,5} & e_{5,6} \\ e_{6,1} & e_{6,2} & e_{6,3} & e_{6,4} & e_{6,5} & e_{6,6} \end{bmatrix} \qquad (3.2)$$

```
> c[1] := Matrix(6,6,[
> [x[1], 0, 0, 0, 0, 0],
> [0, x[2], 0, 0, 0, 0],
> [0, 0, x[3], 0, 0, 0],
> [0, 0, 0, x[4], 0, 0],
> [0, 0, 0, 0, x[5], 0],
> [0, 0, 0, 0, 0, x[6]]]);
```

$$c_1 := \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_6 \end{bmatrix} \qquad (3.3)$$

```
> c[2] := Matrix(6,6,[
> [y[1], 0, 0, 0, 0, 0],
> [0, y[2], 0, 0, 0, 0],
> [0, 0, y[3], 0, 0, 0],
> [0, 0, 0, y[4], 0, 0],
> [0, 0, 0, 0, y[5], 0],
> [0, 0, 0, 0, 0, y[6]]]);
```

$$c_2 := \begin{bmatrix} y_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & y_6 \end{bmatrix} \quad (3.4)$$

```
> Id := Matrix(6, 6, [
> [1, 0, 0, 0, 0, 0],
> [0, 1, 0, 0, 0, 0],
> [0, 0, 1, 0, 0, 0],
> [0, 0, 0, 1, 0, 0],
> [0, 0, 0, 0, 1, 0],
> [0, 0, 0, 0, 0, 1]])
```

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.5)$$

```
> -(x[m] - x[n])^2 + 2*(x[m] + x[n]) +    h[1]*(h[1] +
  2) = 0;
```

$$-(x_m - x_n)^2 + 2\ x_m + 2\ x_n + h_1\ (h_1 + 2) = 0 \quad (3.6)$$

```
> print( UU(Rc5c1[2,1]) );print( UU(Rc5c1[2,3] ));print
  ( UU(Rc5c1[3,4]) );print( UU(Rc5c1[4,5]) );print( UU
  (Rc5c1[5,6]) );
```

$$\frac{(x_1 - x_2)\ (h_1^2 - x_1^2 + 2\ x_2\ x_1 - x_2^2 + 2\ h_1 + 2\ x_1 + 2\ x_2)\ e_{2,1}}{2\ h_1}$$

$$-\frac{(x_2 - x_3)\ (h_1^2 - x_2^2 + 2\ x_2\ x_3 - x_3^2 + 2\ h_1 + 2\ x_2 + 2\ x_3)\ e_{2,3}}{2\ h_1}$$

$$-\frac{(x_3 - x_4)\ (h_1^2 - x_3^2 + 2\ x_3\ x_4 - x_4^2 + 2\ h_1 + 2\ x_3 + 2\ x_4)\ e_{3,4}}{2\ h_1}$$

$$-\frac{(x_4 - x_5)\ (h_1^2 - x_4^2 + 2\ x_4\ x_5 - x_5^2 + 2\ h_1 + 2\ x_4 + 2\ x_5)\ e_{4,5}}{2\ h_1}$$

$$-\frac{(x_5 - x_6)\ (h_1^2 - x_5^2 + 2\ x_5\ x_6 - x_6^2 + 2\ h_1 + 2\ x_5 + 2\ x_6)\ e_{5,6}}{2\ h_1} \quad (3.7)$$

```
> print( UU(Rc5c1[1,3]) );print( UU(Rc5c1[2,5]) );print
  ( UU(Rc5c1[3,6]) );print( UU(Rc5c1[4,1]) );
```

$$-\frac{e_{1,3}\ (x_1 - x_3)\ (h_1^2 - x_1^2 + 2\ x_1\ x_3 - x_3^2 + 2\ h_1 + 2\ x_1 + 2\ x_3)}{2\ h_1}$$

$$-\frac{e_{2,5}\ (x_2 - x_5)\ (h_1^2 - x_2^2 + 2\ x_2\ x_5 - x_5^2 + 2\ h_1 + 2\ x_2 + 2\ x_5)}{2\ h_1}$$

$$-\frac{e_{3,6}\ (x_3 - x_6)\ (h_1^2 - x_3^2 + 2\ x_3\ x_6 - x_6^2 + 2\ h_1 + 2\ x_3 + 2\ x_6)}{2\ h_1}$$

$$\frac{e_{4,1}\ (x_1 - x_4)\ (h_1^2 - x_1^2 + 2\ x_4\ x_1 - x_4^2 + 2\ h_1 + 2\ x_1 + 2\ x_4)}{2\ h_1} \quad (3.8)$$

```
> # It is like A_2; it follows that -->
> # x[i] = (S+i-1)^2-(h[1]+1)^2/4; and      e[i,j] = 0
  for |i-j| > 1;
> for i from 1 to 6 do   x[i] := (S+i-1)^2-(h[1]+1)
  ^2/4; od;
```

$$x_1 := S^2 - \frac{(h_1 + 1)^2}{4}$$

$$x_2 := (S + 1)^2 - \frac{(h_1 + 1)^2}{4}$$

$$x_3 := (S+2)^2 - \frac{(h_1+1)^2}{4}$$

$$x_4 := (S+3)^2 - \frac{(h_1+1)^2}{4}$$

$$x_5 := (S+4)^2 - \frac{(h_1+1)^2}{4}$$

$$x_6 := (S+5)^2 - \frac{(h_1+1)^2}{4} \tag{3.9}$$

```
> for i from 1 to 6 do   for j from 1 to 6 do
>     if (i-j > 1 or j-i > 1) then e[i,j] := 0; fi;
> od; od;
> #-----------------------------------------------------
-----
```

```
> -(y[m] - y[n])^2 + 8*(y[m] + y[n]) + 16*h[3]*(h[3] +
2) = 0;
```

$$-(y_m - y_n)^2 + 8\ y_m + 8\ y_n + 16\ h_3\ (h_3+2) = 0 \tag{3.10}$$

```
> print( UU(Rc7c2[2,1] ));print( UU(Rc7c2[2,3]) );print
( UU(Rc7c2[3,4] ));print( UU(Rc7c2[4,5]) );print( UU
(Rc7c2[5,6]) );
```

$$\frac{(y_1-y_2)\ (16\ h_3^2 - y_1^2 + 2\ y_2\ y_1 - y_2^2 + 32\ h_3 + 8\ y_1 + 8\ y_2)\ e_{2,1}}{16\ h_3}$$

$$-\frac{(y_2-y_3)\ (16\ h_3^2 - y_2^2 + 2\ y_2\ y_3 - y_3^2 + 32\ h_3 + 8\ y_2 + 8\ y_3)\ e_{2,3}}{16\ h_3}$$

$$-\frac{(y_3-y_4)\ (16\ h_3^2 - y_3^2 + 2\ y_3\ y_4 - y_4^2 + 32\ h_3 + 8\ y_3 + 8\ y_4)\ e_{3,4}}{16\ h_3}$$

$$-\frac{(y_4-y_5)\ (16\ h_3^2 - y_4^2 + 2\ y_4\ y_5 - y_5^2 + 32\ h_3 + 8\ y_4 + 8\ y_5)\ e_{4,5}}{16\ h_3}$$

$$-\frac{(y_5-y_6)\ (16\ h_3^2 - y_5^2 + 2\ y_5\ y_6 - y_6^2 + 32\ h_3 + 8\ y_5 + 8\ y_6)\ e_{5,6}}{16\ h_3} \tag{3.11}$$

```
>  for i from 1 to 6 do  y[i] := 4*(T+(i-1))^2-(h[3]+1)
^2;  od;
```

$$y_1 := 4\ T^2 - (h_3+1)^2$$

$$y_2 := 4\ (T+1)^2 - (h_3+1)^2$$

$$y_3 := 4\ (T+2)^2 - (h_3+1)^2$$

$$y_4 := 4\ (T+3)^2 - (h_3+1)^2$$
$$y_5 := 4\ (T+4)^2 - (h_3+1)^2$$
$$y_6 := 4\ (T+5)^2 - (h_3+1)^2 \quad (3.12)$$

```
> UM4(c[1]);
```

$$\begin{bmatrix} \frac{(2\ S+3+h_1)\ (2\ S+1-h_1)}{4} & 0 & \cdots \\ 0 & \frac{(2\ S+5+h_1)\ (2\ S+}{4} & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \end{bmatrix} \quad (3.13)$$

```
> UM4(c[2]);
```

$$\begin{bmatrix} (2\ T+3+h_3)\ (2\ T+1-h_3) & 0 & \cdots \\ 0 & (2\ T+5+h_3)\ (2\ T+ & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \end{bmatrix} \quad (3.14)$$

```
> UM(c[3]);
```

$$\begin{bmatrix} e_{1,1} & e_{1,2} & 0 & 0 & 0 & 0 \\ e_{2,1} & e_{2,2} & e_{2,3} & 0 & 0 & 0 \\ 0 & e_{3,2} & e_{3,3} & e_{3,4} & 0 & 0 \\ 0 & 0 & e_{4,3} & e_{4,4} & e_{4,5} & 0 \\ 0 & 0 & 0 & e_{5,4} & e_{5,5} & e_{5,6} \\ 0 & 0 & 0 & 0 & e_{6,5} & e_{6,6} \end{bmatrix} \quad (3.15)$$

```
> X1 := UU(Rc5c3[3,4]/e[3,4] + Rc5c3[4,3]/e[4,3]);
```

$$X1 := (2\ S+3)\ e_{3,3} - 8\ S^2 - 4\ S\ T - 4\ T^2 + 2\ h_1\ h_3 - 50\ S - 30\ T + 2\ h_1 \quad (3.16)$$

$$-2\ h_3 - 100 + (-2\ S - 7)\ e_{4,4} + z_1$$

```
> X2 := UU(Rc7c3[3,4]/e[3,4] + Rc7c3[4,3]/e[4,3]);
```

$$X2 := (-4\ T - 6)\ e_{3,3} + 8\ S^2 + 8\ S\ T + 16\ T^2 - 4\ h_1\ h_3 + 60\ S + 100\ T - 4\ h_1 + 4\ h_3 + 200 + (4\ T + 14)\ e_{4,4} - 2\ z_1 \quad \textbf{(3.17)}$$

```
> factor(2*X1 + X2);
```

$$-4\ (S - T)\ (2\ S - e_{3,3} + e_{4,4} + 10 + 2\ T) \quad \textbf{(3.18)}$$

```
> # ========= We have two cases: 1. T = S. 2. T <> S;
```

#------ Case 1: T = S---------

```
> SSSS := {z[1],e[1,1],e[2,2],e[3,3],e[4,4],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,3}, e_{3,4}, e_{4,3}, e_{4,4}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(4.1)}$$

```
> T := S; S := a[3]/2; h[1] := a[1]; h[3] := a[1]+a[2]; a[4] := a[3];
```

$$T := S$$

$$S := \frac{a_3}{2}$$

$$h_1 := a_1$$

$$h_3 := a_1 + a_2$$

$$a_4 := a_3 \quad \textbf{(4.2)}$$

```
> X1 := UU(2*h[1]*Rc5c3[3,4]);
```

$$X1 := (a_3 + 3)\ (a_3 + a_1 + 5)\ e_{3,4}\ e_{3,3} - (a_3 + 7)\ (a_3 + a_1 + 5)\ e_{3,4}\ e_{4,4} + (a_3 + a_1 + 5)\ e_{3,4}\ z_1 + 2\ (a_3 + a_1 + 5)\ (a_1^2 + a_1\ a_2 - 2\ a_3^2 - a_2 - 20\ a_3 - 50)\ e_{3,4} \quad \textbf{(4.3)}$$

```
> e[4,4] := UU( solve(X1, e[4,4] ));
```

$$e_{4,4} := \frac{(a_3 + 3)\ e_{3,3}}{a_3 + 7} + \frac{z_1}{a_3 + 7} + \frac{2\ (a_1^2 + a_1\ a_2 - 2\ a_3^2 - a_2 - 20\ a_3 - 50)}{a_3 + 7} \quad \textbf{(4.4)}$$

```
> SSSS := {z[1],e[1,1],e[2,2],e[3,3],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,3}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \tag{4.5}$$

```
> X2 := UU(h[3]*Rc6c5[3,4] );
```

$$X2 := 2\ (a_3+5)\ (a_3+3)\ e_{3,4}\ e_{3,3} + \left(\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_3^2 - 4\ a_3 - \frac{15}{2}\right) e_{3,4}\ z_1 + \left(-a_1^2\ a_3^2 - a_1\ a_2\ a_3^2 + a_3^4 - 8\ a_1^2\ a_3 - 8\ a_1\ a_2\ a_3 + a_2\ a_3^2 + 16\ a_3^3 - 13\ a_1^2 - 13\ a_1\ a_2 + 8\ a_2\ a_3 + 94\ a_3^2 + 15\ a_2 + 240\ a_3 + 225\right) e_{3,4} \tag{4.6}$$

```
> e[3,3] := UU( solve(X2, e[3,3] ));
```

$$e_{3,3} := -\frac{\left(a_1^2 + a_1\ a_2 - a_3^2 - 8\ a_3 - 15\right)\ z_1}{4\ (a_3+5)\ (a_3+3)} + \frac{1}{2\ (a_3+5)\ (a_3+3)}\left(a_1^2\ a_3^2 + a_1\ a_2\ a_3^2 - a_3^4 + 8\ a_1^2\ a_3 + 8\ a_1\ a_2\ a_3 - a_2\ a_3^2 - 16\ a_3^3 + 13\ a_1^2 + 13\ a_1\ a_2 - 8\ a_2\ a_3 - 94\ a_3^2 - 15\ a_2 - 240\ a_3 - 225\right) \tag{4.7}$$

```
> SSSS := {z[1],e[1,1],e[2,2],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \tag{4.8}$$

```
> e[4,4] := UU(e[4,4] );
```

$$e_{4,4} := -\frac{\left(a_1^2 + a_1\ a_2 - a_3^2 - 12\ a_3 - 35\right)\ z_1}{4\ (a_3+7)\ (a_3+5)} + \frac{1}{2\ (a_3+7)\ (a_3+5)}\left(a_1^2\ a_3^2 + a_1\ a_2\ a_3^2 - a_3^4 + 12\ a_1^2\ a_3 + 12\ a_1\ a_2\ a_3 - a_2\ a_3^2 - 24\ a_3^3 + 33\ a_1^2 + 33\ a_1\ a_2 - 12\ a_2\ a_3 - 214\ a_3^2 - 35\ a_2 - 840\ a_3 - 1225\right) \tag{4.9}$$

```
> SSSS := {z[1],e[1,1],e[2,2],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \tag{4.10}$$

```
> e[2,2] := UU( solve(Rc5c3[2,3], e[2,2] ));
```

(4.11)

$$e_{2,2} := -\frac{(a_1^2 + a_1 a_2 - a_3^2 - 4 a_3 - 3) z_1}{4 (a_3+1)(a_3+3)} + \frac{1}{2 (a_3+1)(a_3+3)} (a_1^2 a_3^2 + a_1 a_2 a_3^2 - a_3^4 + 4 a_1^2 a_3 + 4 a_1 a_2 a_3 - a_2 a_3^2 - 8 a_3^3 + a_1^2 + a_1 a_2 - 4 a_2 a_3 - 22 a_3^2 - 3 a_2 - 24 a_3 - 9) \quad (4.11)$$

```
> SSSS := {z[1],e[1,1],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad (4.12)$$

```
> e[1,1] := UU( solve(Rc5c3[1,2], e[1,1] ));
```

$$e_{1,1} := -\frac{(a_1^2 + a_1 a_2 - a_3^2 + 1) z_1}{4 (a_3-1)(a_3+1)} + \frac{a_1^2 a_3^2 + a_1 a_2 a_3^2 - a_3^4 - a_2 a_3^2 - 3 a_1^2 - 3 a_1 a_2 + 2 a_3^2 + a_2 - 1}{2 (a_3-1)(a_3+1)} \quad (4.13)$$

```
> SSSS := {z[1],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad (4.14)$$

```
> e[5,5] := UU( solve(Rc5c3[5,6], e[5,5] ));
```

$$e_{5,5} := \frac{(a_3 + 11) e_{6,6}}{a_3 + 7} - \frac{z_1}{a_3 + 7} - \frac{2 (a_1^2 + a_1 a_2 - 2 a_3^2 - a_2 - 36 a_3 - 162)}{a_3 + 7} \quad (4.15)$$

```
> SSSS := {z[1],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad (4.16)$$

```
> e[6,6] := UU( solve(Rc5c3[4,5], e[6,6] ));
```

$$e_{6,6} := -\frac{(a_1^2 + a_1 a_2 - a_3^2 - 20 a_3 - 99) z_1}{4 (a_3+9)(a_3+11)} + \frac{1}{2 (a_3+9)(a_3+11)} (a_1^2 a_3^2 + a_1 a_2 a_3^2 - a_3^4 + 20 a_1^2 a_3 + 20 a_1 a_2 a_3 - a_2 a_3^2 - 40 a_3^3 + 97 a_1^2 + 97 a_1 a_2 - 20 a_2 a_3 - 598 a_3^2 - 99 a_2 - 3960 a_3 - 9801) \quad (4.17)$$

```
> SSSS := {z[1],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}, z_1\} \quad (4.18)$$

```
> for i from 1 to 6 do  UU( e[i,i]);  od;
```

$$-\frac{(a_1^2 + a_1 a_2 - a_3^2 + 1) z_1}{4 (a_3-1)(a_3+1)} + \frac{a_1^2 a_3^2 + a_1 a_2 a_3^2 - a_3^4 - a_2 a_3^2 - 3 a_1^2 - 3 a_1 a_2 + 2 a_3^2 + a_2 - 1}{2 (a_3-1)(a_3+1)}$$

$$-\frac{(a_1^2+a_1 a_2-a_3^2-4 a_3-3) z_1}{4 (a_3+1) (a_3+3)}+\frac{1}{2 (a_3+1) (a_3+3)}(a_1^2 a_3^2+a_1 a_2 a_3^2-a_3^4+4 a_1^2 a_3+4 a_1 a_2 a_3-a_2 a_3^2-8 a_3^3+a_1^2+a_1 a_2-4 a_2 a_3-22 a_3^2-3 a_2-24 a_3-9)$$

$$-\frac{(a_1^2+a_1 a_2-a_3^2-8 a_3-15) z_1}{4 (a_3+5) (a_3+3)}+\frac{1}{2 (a_3+5) (a_3+3)}(a_1^2 a_3^2+a_1 a_2 a_3^2-a_3^4+8 a_1^2 a_3+8 a_1 a_2 a_3-a_2 a_3^2-16 a_3^3+13 a_1^2+13 a_1 a_2-8 a_2 a_3-94 a_3^2-15 a_2-240 a_3-225)$$

$$-\frac{(a_1^2+a_1 a_2-a_3^2-12 a_3-35) z_1}{4 (a_3+7) (a_3+5)}+\frac{1}{2 (a_3+7) (a_3+5)}(a_1^2 a_3^2+a_1 a_2 a_3^2-a_3^4+12 a_1^2 a_3+12 a_1 a_2 a_3-a_2 a_3^2-24 a_3^3+33 a_1^2+33 a_1 a_2-12 a_2 a_3-214 a_3^2-35 a_2-840 a_3-1225)$$

$$-\frac{(a_1^2+a_1 a_2-a_3^2-16 a_3-63) z_1}{4 (a_3+7) (a_3+9)}+\frac{1}{2 (a_3+7) (a_3+9)}(a_1^2 a_3^2+a_1 a_2 a_3^2-a_3^4+16 a_1^2 a_3+16 a_1 a_2 a_3-a_2 a_3^2-32 a_3^3+61 a_1^2+61 a_1 a_2-16 a_2 a_3-382 a_3^2-63 a_2-2016 a_3-3969)$$

$$-\frac{(a_1^2+a_1 a_2-a_3^2-20 a_3-99) z_1}{4 (a_3+9) (a_3+11)}+\frac{1}{2 (a_3+9) (a_3+11)}(a_1^2 a_3^2+a_1 a_2 a_3^2-a_3^4+20 a_1^2 a_3+20 a_1 a_2 a_3-a_2 a_3^2-40 a_3^3+97 a_1^2+97 a_1 a_2-20 a_2 a_3-598 a_3^2-99 a_2-3960 a_3-9801) \quad \textbf{(4.19)}$$

```
> X1 := UU((-h[1] -1 + 2*S)*Rc5c3[2,2] + 2*Rc5c6[2,2] )
  ;
```

$$X1 := 2 a_3 (a_3+2) e_{2,1} e_{1,2} - \frac{(a_3+a_1+1) (a_1-1-a_3) (a_3+1+a_2+a_1) (-a_3-1+a_2+a_1) z_1^2}{32 (a_3+1)^2} + \frac{1}{8 (a_3+1)^2}((a_3^2+2 a_3-1) (a_3+a_1+1) (a_1-1-a_3) (a_3+1+a_2+a_1) (-a_3-1+a_2+a_1) z_1) - \frac{1}{8 (a_3+1)^2}(a_3 (a_3-1) (a_3+3) (a_3+2) (a_3+a_1+1) (a_1-1-a_3) (a_3+1+a_2+a_1) (-a_3-1+a_2+a_1)) \quad \textbf{(4.20)}$$

```
> X2 := UU((-h[1] + 1 + 2*S)*Rc5c3[3,3] + 2*Rc5c6[3,3]
  );
```

$$X2 := 2 (a_3+4) (a_3+2) e_{3,2} e_{2,3} \quad \textbf{(4.21)}$$

$$-\frac{1}{32\,(a_3+3)^2}\big((a_3+a_1+3)\,(-a_3+a_1-3)\,(a_3+3+a_2+a_1)\,(-a_3-3+a_2+a_1)\,z_1^2\big)+\frac{1}{8\,(a_3+3)^2}\big((a_3^2+6\,a_3+7)\,(a_3+a_1+3)\,(-a_3+a_1-3)\,(a_3+3+a_2+a_1)\,(-a_3-3+a_2+a_1)\,z_1\big)-\frac{1}{8\,(a_3+3)^2}\big((a_3+5)\,(a_3+4)\,(a_3+2)\,(a_3+1)\,(a_3+a_1+3)\,(-a_3+a_1-3)\,(a_3+3+a_2+a_1)\,(-a_3-3+a_2+a_1)\big)$$

```
> X3 := UU((-h[1] + 3 + 2*S)*Rc5c3[4,4] + 2*Rc5c6[4,4]
  );
```

$$X3 := 2\,(a_3+6)\,(a_3+4)\,e_{4,3}\,e_{3,4}-\frac{1}{32\,(a_3+5)^2}\big((a_3+a_1+5)\,(-a_3+a_1-5)\,(a_3+5+a_2+a_1)\,(-a_3-5+a_2+a_1)\,z_1^2\big)+\frac{1}{8\,(a_3+5)^2}\big((a_3^2+10\,a_3+23)\,(a_3+a_1+5)\,(-a_3+a_1-5)\,(a_3+5+a_2+a_1)\,(-a_3-5+a_2+a_1)\,z_1\big)-\frac{1}{8\,(a_3+5)^2}\big((a_3+4)\,(a_3+3)\,(a_3+7)\,(a_3+6)\,(a_3+a_1+5)\,(-a_3+a_1-5)\,(a_3+5+a_2+a_1)\,(-a_3-5+a_2+a_1)\big) \quad \textbf{(4.22)}$$

```
> X4 := UU((-h[1] + 5 + 2*S)*Rc5c3[5,5] + 2*Rc5c6[5,5]
  );
```

$$X4 := 2\,(a_3+8)\,(a_3+6)\,e_{5,4}\,e_{4,5}-\frac{(a_1+7+a_3)\,(a_1-7-a_3)\,(a_3+7+a_2+a_1)\,(-a_3-7+a_2+a_1)\,z_1^2}{32\,(a_3+7)^2}+\frac{1}{8\,(a_3+7)^2}\big((a_3^2+14\,a_3+47)\,(a_1+7+a_3)\,(a_1-7-a_3)\,(a_3+7+a_2+a_1)\,(-a_3-7+a_2+a_1)\,z_1\big)-\frac{1}{8\,(a_3+7)^2}\big((a_3+5)\,(a_3+9)\,(a_3+8)\,(a_3+6)\,(a_1+7+a_3)\,(a_1-7-a_3)\,(a_3+7+a_2+a_1)\,(-a_3-7+a_2+a_1)\big) \quad \textbf{(4.23)}$$

```
> e[2,1] := solve(X1, e[2,1]);
```

$$e_{2,1} := \frac{1}{64\,(a_3+1)^2\,a_3\,(a_3+2)\,e_{1,2}}\big((a_3+a_1+1)\,(a_1-1-a_3)\,(a_3+1+a_2+a_1)\,(-a_3-1+a_2+a_1)\,(4\,a_3^4+16\,a_3^3-4\,a_3^2\,z_1+4\,a_3^2-8\,a_3\,z_1+z_1^2-24\,a_3+4\,z_1)\big) \quad \textbf{(4.24)}$$

```
> e[3,2] := solve(X2, e[3,2]);
```

(4.25)

$$e_{3,2} := \frac{1}{64\ (a_3+3)^2\ (a_3+4)\ (a_3+2)\ e_{2,3}} \big((a_3+a_1+3)\ (-a_3+a_1-3)\ (a_3+3+a_2+a_1)\ (-a_3-3+a_2+a_1)\ (4\ a_3^4+48\ a_3^3-4\ a_3^2\ z_1+196\ a_3^2-24\ a_3\ z_1+z_1^2+312\ a_3-28\ z_1+160)\big) \quad \textbf{(4.25)}$$

```
> e[4,3] := solve(X3, e[4,3]);
```

$$e_{4,3} := \frac{1}{64\ (a_3+5)^2\ (a_3+6)\ (a_3+4)\ e_{3,4}} \big((a_3+a_1+5)\ (-a_3+a_1-5)\ (a_3+5+a_2+a_1)\ (-a_3-5+a_2+a_1)\ (4\ a_3^4+80\ a_3^3-4\ a_3^2\ z_1+580\ a_3^2-40\ a_3\ z_1+z_1^2+1800\ a_3-92\ z_1+2016)\big) \quad \textbf{(4.26)}$$

```
> e[5,4] := solve(X4, e[5,4]);
```

$$e_{5,4} := \frac{1}{64\ (a_3+7)^2\ (a_3+8)\ (a_3+6)\ e_{4,5}} \big((a_1+7+a_3)\ (a_1-7-a_3)\ (a_3+7+a_2+a_1)\ (-a_3-7+a_2+a_1)\ (4\ a_3^4+112\ a_3^3-4\ a_3^2\ z_1+1156\ a_3^2-56\ a_3\ z_1+z_1^2+5208\ a_3-188\ z_1+8640)\big) \quad \textbf{(4.27)}$$

```
> SSSS := {e[1,2],e[2,3],e[3,4],e[4,5],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,6}, e_{6,5}\} \quad \textbf{(4.28)}$$

```
> e[6,5] := UU(solve(Rc5c3[5,5], e[6,5]) );
```

$$e_{6,5} := \frac{1}{64\ (a_3+9)^2\ (a_3+8)\ (a_3+10)\ e_{5,6}} \big((2\ a_3^2+34\ a_3-z_1+140)\ (2\ a_3^2+38\ a_3-z_1+176)\ (a_1+9+a_3)\ (a_1-9-a_3)\ (a_3+9+a_2+a_1)\ (-a_3-9+a_2+a_1)\big) \quad \textbf{(4.29)}$$

```
> SSSS := {e[1,2],e[2,3],e[3,4],e[4,5],e[5,6]};
```

$$SSSS := \{e_{1,2}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,6}\} \quad \textbf{(4.30)}$$

```
> z[1] := 2*(nu+1)*(nu-2);
```

$$z_1 := 2\ (\nu+1)\ (\nu-2) \quad \textbf{(4.31)}$$

```
> UM4(Rc5c3);UM4(Rc7c2);UM4(Rc7c3);UM4(Rc5c6);UM4(Rc6c5);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(4.32)}$$

```
> UM4(c[1]);UM4(c[2]);UM4(c[3]);
```

$$\begin{bmatrix} -\frac{(a_3 + a_1 + 3)\ (a_1 - 1 - a_3)}{4} & 0 & \cdots \\ 0 & -\frac{(a_3 + a_1 + 5)\ (-a_3 +}{4} & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

$$\begin{bmatrix} -(a_3+3+a_2+a_1)\ (-a_3-1+a_2+a_1) & \cdots \\ 0 \quad -(a_3+\text{!} & \cdots \\ 0 & \cdots \\ 0 & \cdots \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\nu^2\ a_1^2+\nu^2\ a_1\ a_2-\nu^2\ a_3^2-a_1^2\ a_3^2-a_1}{} & \cdots \\ & \cdots \\ & \cdots \\ & \cdots \end{bmatrix} \quad \textbf{(4.33)}$$

```
> # All relations are resolved. We can  set any values
  to e[ii,ii+1]. Set it match them to the "article"
```

```
> # Using formulas from the artivcle create 6·6 matrices
    and compare them with matrices received after
    resolving relations.
> #
> SSSS := {};
```

$$SSSS := \varnothing \quad \textbf{(4.34)}$$

```
> #UM(c[1]);UM(c[2]);
> #UM4(c[3]);
```

```
> i := 'i'; j := 'j'; k := 'k';s := 's';t := 't';
```

$$i := i$$
$$j := j$$
$$k := k$$
$$s := s$$

$$t := t \quad (4.35)$$

```
> H01 := (i,j) -> ( a[1] + 2*i    -j );
```

$$H01 := (i, j) \mapsto a_1 + 2 \cdot i - j \quad (4.36)$$

```
> H10 := (i,j) -> ( a[2] - 2*i + 2*j );
```

$$H10 := (i, j) \mapsto a_2 - 2 \cdot i + 2 \cdot j \quad (4.37)$$

```
> H2  := (i,j) -> H01(i,j) + H10(i,j); # ( j +h[1] + h
[10])
```

$$H2 := (i, j) \mapsto H01(i, j) + H10(i, j) \quad (4.38)$$

```
> s   := (j,k) -> (a[3] -j + 2*k - 1 );
```

$$s := (j, k) \mapsto a_3 - j + 2 \cdot k - 1 \quad (4.39)$$

```
> #t   := (j,k) -> (a[4] -j + 2*k - 1 );
> Sp  := (i,j,k) -> ( a[1] + a[3] + 2*i  - 2*j + 2*k -
1  )/2;
```

$$Sp := (i, j, k) \mapsto \frac{a_1}{2} + \frac{a_3}{2} + i - j + k - \frac{1}{2} \quad (4.40)$$

```
> Sm  := (i,k)   -> (-a[1] + a[3] - 2*i        + 2*k -
1  )/2;
```

$$Sm := (i, k) \mapsto -\frac{a_1}{2} + \frac{a_3}{2} - i + k - \frac{1}{2} \quad (4.41)$$

```
> Tp  := (k)     -> ( a[1] + a[2] + a[4]       + 2*k -
1  )/2;
```

$$Tp := k \mapsto \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_4}{2} + k - \frac{1}{2} \quad (4.42)$$

```
> Tm  := (j,k)   -> (-a[1] - a[2] + a[4] - 2*j + 2*k -
1  )/2;
```

$$Tm := (j, k) \mapsto -\frac{a_1}{2} - \frac{a_2}{2} + \frac{a_4}{2} - j + k - \frac{1}{2} \quad (4.43)$$

```
> Qp  := (j,k)   -> nu / s(j,k) + 1;
```

$$Qp := (j, k) \mapsto \frac{\nu}{s(j, k)} + 1 \quad (4.44)$$

```
> Qm  := (j,k)   -> nu / s(j,k) - 1;
```

$$Qm := (j, k) \mapsto \frac{\nu}{s(j, k)} - 1 \quad (4.45)$$

```
> nu;
```

$$\nu \quad (4.46)$$

```
> ME01 := (i,j,k) -> Matrix( 6,6,[
> [          Sp(i,j,k  ),0,0,0,0,0],
> [0,        Sp(i,j,k+1),0,0,0,0],
> [0,0,      Sp(i,j,k+2),0,0,0],
> [0,0,0,    Sp(i,j,k+3),0,0],
> [0,0,0,0,  Sp(i,j,k+4),0],
> [0,0,0,0,0,Sp(i,j,k+5)]]);
```

$$ME01 := (i, j, k) \mapsto Matrix(6, 6, [[Sp(i, j, k), 0, 0, 0, 0, 0], [0, Sp(i, j, k+1), 0, 0, 0, 0], [0, 0, Sp(i, j, k+2), 0, 0, 0], [0, 0, 0, Sp(i, j, k+3), 0, 0], [0, 0, 0, 0, Sp(i, j, k+4), 0], [0, 0, 0, 0, 0, Sp(i, j, k+5)]]) \quad (4.47)$$

```
> MF01 := (i,j,k) -> Matrix( 6,6,[
> [          Sm(i,k  ),0,0,0,0,0],
> [0,        Sm(i,k+1),0,0,0,0],
> [0,0,      Sm(i,k+2),0,0,0],
> [0,0,0,    Sm(i,k+3),0,0],
> [0,0,0,0,  Sm(i,k+4),0],
> [0,0,0,0,0,Sm(i,k+5)]]);
```

$$MF01 := (i, j, k) \mapsto Matrix(6, 6, [[Sm(i, k), 0, 0, 0, 0, 0], [0, Sm(i, k+1), 0, 0, 0, 0], [0, 0, Sm(i, k+2), 0, 0, 0], [0, 0, 0, Sm(i, k+3), 0, 0], [0, 0, 0, 0, Sm(i, k+4), 0], [0, 0, 0, 0, 0, Sm(i, k+5)]]) \quad (4.48)$$

```
> MH1 := UM(ME01(-1,0,k) . MF01(0,0,k) - MF01(1,0,k) . ME01(0,0,k));
```

$$MH1 := \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 \end{bmatrix} \quad (4.49)$$

```
> UM( c[1] - MF01(1,0,1).ME01(0,0,1));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(4.50)}$$

```
> ME21 := (i,j,k) -> Matrix( 6,6,[
> [           2*Tm(j+1,k  ),0,0,0,0,0],
> [0,         2*Tm(j+1,k+1),0,0,0,0],
> [0,0,       2*Tm(j+1,k+2),0,0,0],
> [0,0,0,     2*Tm(j+1,k+3),0,0],
> [0,0,0,0,   2*Tm(j+1,k+4),0],
> [0,0,0,0,0,2*Tm(j+1,k+5)]]);
```

$ME21 := (i, j, k) \mapsto Matrix(6, 6, [[2 \cdot Tm(j+1, k), 0, 0, 0, 0, 0], [0, 2 \cdot Tm(j+1, k+1), 0, 0, 0, 0], [0, 0, 2 \cdot Tm(j+1, k+2), 0, 0, 0], [0, 0, 0, 2 \cdot Tm(j+1, k+3), 0, 0], [0, 0, 0, 0, 2 \cdot Tm(j+1, k+4), 0], [0, 0, 0, 0, 0, 2 \cdot Tm(j+1, k+5)]])$ **(4.51)**

```
> MF21 := (i,j,k) -> Matrix( 6,6,[
> [           2*Tp(k-1),0,0,0,0,0],
> [0,         2*Tp(k  ),0,0,0,0],
> [0,0,       2*Tp(k+1),0,0,0],
> [0,0,0,     2*Tp(k+2),0,0],
> [0,0,0,0,   2*Tp(k+3),0],
> [0,0,0,0,0,2*Tp(k+4)]]);
```

$MF21 := (i, j, k) \mapsto Matrix(6, 6, [[2 \cdot Tp(k-1), 0, 0, 0, 0, 0], [0, 2 \cdot Tp(k), 0, 0, 0, 0], [0, 0, 2 \cdot Tp(k+1), 0, 0, 0], [0, 0, 0, 2 \cdot Tp(k+2), 0, 0], [0, 0, 0, 0, 2 \cdot Tp(k+3), 0], [0, 0, 0, 0, 0, 2 \cdot Tp(k+4)]])$ **(4.52)**

```
> UM(c[2]);
```

(4.53)

$$\begin{bmatrix} -(a_3+1+a_2+a_1)\ (-a_3+1+a_2+a_1) & \cdots \\ 0 \qquad -(a_3+ & \cdots \\ 0 & \cdots \\ 0 & \cdots \\ 0 & \cdots \\ 0 & \cdots \end{bmatrix} \quad (4.53)$$

```
> UM(MF21(1,2,1+1) . ME21(0,0,1) - c[2]);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.54)$$

```
> MH21 := UM(ME21(-1,-2,k-1) . MF21(0,0,k) - MF21(1,2,
  k+1) . ME21(0,0,k));
```

$MH21 :=$ (4.55)

$$\begin{bmatrix} 4\ a_1+4\ a_2 & 0 & 0 & 0 & \cdots \\ 0 & 4\ a_1+4\ a_2 & 0 & 0 & \cdots \\ 0 & 0 & 4\ a_1+4\ a_2 & 0 & \cdots \\ 0 & 0 & 0 & 4\ a_1+4\ a_2 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \end{bmatrix}$$

```
> ME10 := (i,j,k) -> Matrix( 6,6,[
> [                                    Tm(j+1,k  )*Qm(j+1,k
```

```
), 0,0,0,0,0 ],
> [         Sm(i,k  )*Qp(j+1,k  ), Tm(j+1,k+1)*Qm(j+1,
  k+1), 0,0,0,0 ],
> [0,       Sm(i,k+1)*Qp(j+1,k+1), Tm(j+1,k+2)*Qm(j+1,
  k+2), 0,0,0 ],
> [0,0,     Sm(i,k+2)*Qp(j+1,k+2), Tm(j+1,k+3)*Qm(j+1,
  k+3), 0,0 ],
> [0,0,0,   Sm(i,k+3)*Qp(j+1,k+3), Tm(j+1,k+4)*Qm(j+1,
  k+4), 0 ],
> [0,0,0,0, Sm(i,k+4)*Qp(j+1,k+4), Tm(j+1,k+5)*Qm(j+1,
  k+5) ]]);
```

$$ME10 := (i, j, k) \mapsto Matrix(6, 6, [[Tm(j+1, k) \cdot Qm(j+1, k), 0, 0, 0, 0, 0], [Sm(i, k) \cdot Qp(j+1, k), Tm(j+1, k+1) \cdot Qm(j+1, k+1), 0, 0, 0, 0], [0, Sm(i, k+1) \cdot Qp(j+1, k+1), Tm(j+1, k+2) \cdot Qm(j+1, k+2), 0, 0, 0], [0, 0, Sm(i, k+2) \cdot Qp(j+1, k+2), Tm(j+1, k+3) \cdot Qm(j+1, k+3), 0, 0], [0, 0, 0, Sm(i, k+3) \cdot Qp(j+1, k+3), Tm(j+1, k+4) \cdot Qm(j+1, k+4), 0], [0, 0, 0, 0, Sm(i, k+4) \cdot Qp(j+1, k+4), Tm(j+1, k+5) \cdot Qm(j+1, k+5)]]) \quad (4.56)$$

```
> MF10 := (i,j,k) -> Matrix( 6,6,[
> [           Sp(i,j,k  )*Qp(j+1,k  ), Tp(k  )*Qm(j+1,
  k+1),0,0,0,0],
> [0,         Sp(i,j,k+1)*Qp(j+1,k+1), Tp(k+1)*Qm(j+1,
  k+2),0,0,0],
> [0,0,       Sp(i,j,k+2)*Qp(j+1,k+2), Tp(k+2)*Qm(j+1,
  k+3),0,0],
> [0,0,0,     Sp(i,j,k+3)*Qp(j+1,k+3), Tp(k+3)*Qm(j+1,
  k+4),0],
> [0,0,0,0,   Sp(i,j,k+4)*Qp(j+1,k+4), Tp(k+4)*Qm(j+1,
  k+5)],
> [0,0,0,0,0,Sp(i,j,k+5)*Qp(j+1,k+5)  ]]);
```

$$MF10 := (i, j, k) \mapsto Matrix(6, 6, [[Sp(i, j, k) \cdot Qp(j+1, k), Tp(k) \cdot Qm(j+1, k+1), 0, 0, 0, 0], [0, Sp(i, j, k+1) \cdot Qp(j+1, k+1), Tp(k+1) \cdot Qm(j+1, k+2), 0, 0, 0], [0, 0, Sp(i, j, k+2) \cdot Qp(j+1, k+2), Tp(k+2) \cdot Qm(j+1, k+3), 0, 0], [0, 0, 0, Sp(i, j, k+3) \cdot Qp(j+1, k+3), Tp(k+3) \cdot Qm(j+1, k+4), 0], [0, 0, 0, 0, Sp(i, j, k+4) \cdot Qp(j+1, k+4), Tp(k+4) \cdot Qm(j+1, k+5)], [0, 0, 0, 0, 0, Sp(i, j, k+5) \quad (4.57)$$

$\cdot Qp(j+1, k+5)]])$

```
> for ii from 1 to 5 do e[ii,ii+1] := UU( Tp(ii) * Tm(0,
  ii) * Qm(0,ii) * Qm(1,ii+1)); od;
```

$$e_{1,2} := -\frac{(a_3+1+a_2+a_1)\,(-a_3-1+a_2+a_1)\,(\nu-a_3-1)\,(\nu-a_3-2)}{4\,(a_3+1)\,(a_3+2)}$$

$$e_{2,3} := -\frac{(a_3+3+a_2+a_1)\,(-a_3-3+a_2+a_1)\,(\nu-3-a_3)\,(\nu-4-a_3)}{4\,(a_3+3)\,(a_3+4)}$$

$$e_{3,4} := -\frac{(a_3+5+a_2+a_1)\,(-a_3-5+a_2+a_1)\,(\nu-5-a_3)\,(\nu-6-a_3)}{4\,(a_3+5)\,(a_3+6)}$$

$$e_{4,5} := -\frac{(a_3+7+a_2+a_1)\,(-a_3-7+a_2+a_1)\,(\nu-7-a_3)\,(\nu-8-a_3)}{4\,(a_3+7)\,(a_3+8)}$$

$$e_{5,6} := -\frac{(a_3+9+a_2+a_1)\,(-a_3-9+a_2+a_1)\,(\nu-a_3-9)\,(\nu-a_3-10)}{4\,(a_3+9)\,(a_3+10)} \tag{4.58}$$

```
> MH10 := UM4(ME10(0,-1,k) . MF10(0,0,k) - MF10(0,1,k)
  . ME10(0,0,k));
```

$$MH10 := \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & a_2 \end{bmatrix} \tag{4.59}$$

```
> UM4( c[3] - MF10(0,1,1).ME10(0,0,1));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.60}$$

```
#  Case 2: T <> S --------------------------
> SSSS := {z[1],e[1,1],e[2,2],e[3,3],e[4,4],e[5,5],e[6,
```

```
6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e
[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, e_{2,3}, e_{3,2}, e_{3,3}, e_{3,4}, e_{4,3}, e_{4,4}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad (5.1)$$

```
> h[3] := a[1]+a[2];
```

$$h_3 := a_1 + a_2 \quad (5.2)$$

```
> T := a[4]/2; S := a[3]/2; h[1] := a[1]; h[10] := a[2]
;
```

$$T := \frac{a_4}{2}$$

$$S := \frac{a_3}{2}$$

$$h_1 := a_1$$

$$h_{10} := a_2 \quad (5.3)$$

```
> #Rc5c3 := UM(Rc5c3);
```

```
> X1 := UU( Rc5c3[3,2]/e[3, 2] ); X2 := UU( Rc5c3[2,3]
/e[2, 3] );
```

$$X1 := \frac{(a_3+1)\ (-a_3+a_1-3)\ e_{2,2}}{2\ a_1} - \frac{(a_3+5)\ (-a_3+a_1-3)\ e_{3,3}}{2\ a_1} + \frac{(-a_3+a_1-3)\ z_1}{2\ a_1} + \frac{1}{2\ a_1}((-a_3+a_1-3)\ (2\ a_1^2+2\ a_1\ a_2-2\ a_3^2 - a_3\ a_4 - a_4^2 - 2\ a_2 - 15\ a_3 - 9\ a_4 - 36))$$

$$X2 := \frac{(a_3+1)\ (a_3+a_1+3)\ e_{2,2}}{2\ a_1} - \frac{(a_3+5)\ (a_3+a_1+3)\ e_{3,3}}{2\ a_1} + \frac{(a_3+a_1+3)\ z_1}{2\ a_1} + \frac{1}{2\ a_1}((a_3+a_1+3)\ (2\ a_1^2+2\ a_1\ a_2-2\ a_3^2-a_3\ a_4-a_4^2-2\ a_2 - 15\ a_3 - 9\ a_4 - 36)) \quad (5.4)$$

```
> Y1 := UU( h[3]*Rc7c3[3,2]/e[3, 2] ); Y2 := UU( h[3]*
Rc7c3[2,3]/e[2, 3] );
```

$$Y1 := -(a_4+1)\ (a_4+a_1+a_2+3)\ e_{2,2} + (a_4+5)\ (a_4+a_1+a_2+3)\ e_{3,3} + (-a_4-a_1-a_2-3)\ z_1 - (a_4+a_1+a_2+3)\ (2\ a_1^2+2\ a_1\ a_2 - a_3^2 - a_3\ a_4 - 2\ a_4^2 - 2\ a_2 - 9\ a_3 - 15\ a_4 - 36)$$

(5.5)

$$Y2 := -(a_4+1)(-a_4+a_1+a_2-3)\, e_{2,2} + (a_4+5)(-a_4+a_1+a_2-3)\, e_{3,3} + (a_4-a_1-a_2+3)\, z_1 - (-a_4+a_1+a_2-3)(2\,a_1^2+2\,a_1\,a_2-a_3^2-a_3\,a_4 - 2\,a_4^2-2\,a_2-9\,a_3-15\,a_4-36) \quad \textbf{(5.5)}$$

```
> e[3,3] := solve(X1, e[3,3]);
```

$$e_{3,3} := \frac{1}{a_3+5}(2\,a_1^2+2\,a_1\,a_2-2\,a_3^2-a_3\,a_4+a_3\,e_{2,2}-a_4^2-2\,a_2-15\,a_3 - 9\,a_4+e_{2,2}+z_1-36) \quad \textbf{(5.6)}$$

```
> e[2,2] := solve(Y1, e[2,2]);
```

$$e_{2,2} := \frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{4}\,a_3^2-\frac{1}{4}\,a_4^2-\frac{1}{2}\,a_2-a_3-a_4+\frac{1}{4}\,z_1-\frac{3}{2} \quad \textbf{(5.7)}$$

```
> SSSS := {z[1],e[1,1],e[4,4],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,4}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(5.8)}$$

```
> e[1,1] := solve(Rc5c3[2,1],e[1,1]);
```

$$e_{1,1} := \frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{4}\,a_3^2-\frac{1}{4}\,a_4^2-\frac{1}{2}\,a_2+\frac{1}{4}\,z_1+\frac{1}{2} \quad \textbf{(5.9)}$$

```
> SSSS := {z[1],e[4,4],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,4}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(5.10)}$$

```
> e[4,4] := solve(Rc5c3[4,3],e[4,4]);
```

$$e_{4,4} := \frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{4}\,a_3^2-\frac{1}{4}\,a_4^2-\frac{1}{2}\,a_2-3\,a_3-3\,a_4+\frac{1}{4}\,z_1-\frac{35}{2} \quad \textbf{(5.11)}$$

```
> SSSS := {z[1],e[5,5],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,5}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(5.12)}$$

```
> e[5,5] := solve(Rc5c3[5,4],e[5,5]);
```

$$e_{5,5} := \frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{4}\,a_3^2-\frac{1}{4}\,a_4^2-\frac{1}{2}\,a_2-4\,a_3-4\,a_4+\frac{1}{4}\,z_1-\frac{63}{2} \quad \textbf{(5.13)}$$

```
> SSSS := {z[1],e[6,6],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}, e_{6,6}, z_1\} \quad \textbf{(5.14)}$$

```
> e[6,6] := solve(Rc5c3[6,5],e[6,6]);
```

$$e_{6,6} := \frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{4}\ a_3^2 - \frac{1}{4}\ a_4^2 - \frac{1}{2}\ a_2 - 5\ a_3 - 5\ a_4 + \frac{1}{4}\ z_1 - \frac{99}{2} \quad (5.15)$$

```
> SSSS := {z[1],e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,
3],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}, z_1\} \quad (5.16)$$

```
> z[1] := factor(solve( Rc6c5[3,2], z[1] ));
```

$$z_1 := (a_3 - a_4 + 2)\ (a_3 - a_4 - 2) \quad (5.17)$$

```
> SSSS := {e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,3],e
[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,3}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}\} \quad (5.18)$$

```
> for ii from 1 to 6 do UU(e[ii,ii]); od;
```

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_2 - \frac{1}{2}\ a_3\ a_4 - \frac{1}{2}$$

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_2 - a_3 - a_4 - \frac{1}{2}\ a_3\ a_4 - \frac{5}{2}$$

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_3\ a_4 - \frac{1}{2}\ a_2 - 2\ a_3 - 2\ a_4 - \frac{17}{2}$$

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_2 - 3\ a_3 - 3\ a_4 - \frac{1}{2}\ a_3\ a_4 - \frac{37}{2}$$

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_2 - 4\ a_3 - 4\ a_4 - \frac{1}{2}\ a_3\ a_4 - \frac{65}{2}$$

$$\frac{1}{2}\ a_1^2 + \frac{1}{2}\ a_1\ a_2 - \frac{1}{2}\ a_2 - 5\ a_3 - 5\ a_4 - \frac{1}{2}\ a_3\ a_4 - \frac{101}{2} \quad (5.19)$$

```
> e[4,3] := UU(solve(Rc5c3[3,3],e[4,3]));
```

$$e_{4,3} := \frac{(a_3+2)\ e_{2,3}\ e_{3,2}}{(a_3+6)\ e_{3,4}} + \frac{1}{4\ e_{3,4}\ (a_3+6)}\big(a_1^4 + 2\ a_1^3\ a_2 + a_1^2\ a_2^2 - 2\ a_1^2\ a_3^2 - a_1^2\ a_3\ a_4 - a_1^2\ a_4^2 - 4\ a_1\ a_2\ a_3^2 - 2\ a_2^2\ a_3^2 + a_3^3\ a_4 + 2\ a_3^2\ a_4^2 - 20\ a_3\ a_1^2 - 12\ a_1^2\ a_4 - 32\ a_3\ a_1\ a_2 - 16\ a_2^2\ a_3 + 4\ a_3^3 + 28\ a_3^2\ a_4 + 16\ a_3\ a_4^2 - 66\ a_1^2 - 66\ a_1\ a_2 - 33\ a_2^2 + 82\ a_3^2 + 181\ a_3\ a_4 + 33\ a_4^2 + 484\ a_3 + 348\ a_4 + 897\big) \quad (5.20)$$

```
> SSSS := {e[1,2],e[2,1],e[2,3],e[3,2],e[3,4],e[4,5],e
[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,2}, e_{3,4}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}\} \quad (5.21)$$

```
> e[3,2] := UU(solve(Rc7c3[3,3],e[3,2]));
```

(5.22)

$$e_{3,2} := \frac{(a_3+a_1+3)\,(-a_3+a_1-3)\,(a_4+a_1+a_2+3)\,(-a_4+a_1+a_2-3)}{16\,e_{2,3}} \quad \textbf{(5.22)}$$

```
> SSSS := {e[1,2],e[2,1],e[2,3],e[3,4],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,1}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}\} \quad \textbf{(5.23)}$$

```
> e[2,1] := UU(solve(Rc5c3[2,2],e[2,1]));
```

$$e_{2,1} := \frac{(a_1+1+a_3)\,(a_1-1-a_3)\,(a_4+1+a_2+a_1)\,(-a_4-1+a_2+a_1)}{16\,e_{1,2}} \quad \textbf{(5.24)}$$

```
> SSSS := {e[1,2],e[2,3],e[3,4],e[4,5],e[5,4],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,4}, e_{5,6}, e_{6,5}\} \quad \textbf{(5.25)}$$

```
> e[5,4] := UU(solve(Rc5c3[4,4],e[5,4]));
```

$$e_{5,4} := \frac{(a_1+7+a_3)\,(a_1-7-a_3)\,(a_4+7+a_2+a_1)\,(-a_4-7+a_2+a_1)}{16\,e_{4,5}} \quad \textbf{(5.26)}$$

```
> SSSS := {e[1,2],e[2,3],e[3,4],e[4,5],e[5,6],e[6,5]};
```

$$SSSS := \{e_{1,2}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,6}, e_{6,5}\} \quad \textbf{(5.27)}$$

```
> e[6,5] := UU(solve(Rc5c3[5,5],e[6,5]));
```

$$e_{6,5} := \frac{(a_1+9+a_3)\,(a_1-9-a_3)\,(a_4+9+a_2+a_1)\,(-a_4-9+a_2+a_1)}{16\,e_{5,6}} \quad \textbf{(5.28)}$$

```
> SSSS := {e[1,2],e[2,3],e[3,4],e[4,5],e[5,6]};
```

$$SSSS := \{e_{1,2}, e_{2,3}, e_{3,4}, e_{4,5}, e_{5,6}\} \quad \textbf{(5.29)}$$

```
> SSSS := {};
```

$$SSSS := \varnothing \quad \textbf{(5.30)}$$

```
> for ii from 1 to 5 do UU(e[ii+1,ii]);  UU(e[ii,ii]);
  UU(e[ii,ii+1]); od;
```

$$\frac{(a_1+1+a_3)\,(a_1-1-a_3)\,(a_4+1+a_2+a_1)\,(-a_4-1+a_2+a_1)}{16\,e_{1,2}}$$

$$\frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{2}\,a_2-\frac{1}{2}\,a_3\,a_4-\frac{1}{2}$$

$$e_{1,2}$$

$$\frac{(a_3+a_1+3)\,(-a_3+a_1-3)\,(a_4+a_1+a_2+3)\,(-a_4+a_1+a_2-3)}{16\,e_{2,3}}$$

$$\frac{1}{2}\,a_1^2+\frac{1}{2}\,a_1\,a_2-\frac{1}{2}\,a_2-a_3-a_4-\frac{1}{2}\,a_3\,a_4-\frac{5}{2}$$

$$e_{2,3}$$

$$\frac{(a_1+5+a_3)\ (a_1-5-a_3)\ (a_4+5+a_2+a_1)\ (-a_4-5+a_2+a_1)}{16\ e_{3,4}}$$

$$\frac{\frac{1}{2}\ a_1^2+\frac{1}{2}\ a_1\ a_2-\frac{1}{2}\ a_3\ a_4-\frac{1}{2}\ a_2-2\ a_3-2\ a_4-\frac{17}{2}}{e_{3,4}}$$

$$\frac{(a_1+7+a_3)\ (a_1-7-a_3)\ (a_4+7+a_2+a_1)\ (-a_4-7+a_2+a_1)}{16\ e_{4,5}}$$

$$\frac{\frac{1}{2}\ a_1^2+\frac{1}{2}\ a_1\ a_2-\frac{1}{2}\ a_2-3\ a_3-3\ a_4-\frac{1}{2}\ a_3\ a_4-\frac{37}{2}}{e_{4,5}}$$

$$\frac{(a_1+9+a_3)\ (a_1-9-a_3)\ (a_4+9+a_2+a_1)\ (-a_4-9+a_2+a_1)}{16\ e_{5,6}}$$

$$\frac{\frac{1}{2}\ a_1^2+\frac{1}{2}\ a_1\ a_2-\frac{1}{2}\ a_2-4\ a_3-4\ a_4-\frac{1}{2}\ a_3\ a_4-\frac{65}{2}}{e_{5,6}}$$

(5.31)

```
> nu := 0;
```

$$\nu := 0$$

(5.32)

```
> UM4( Rc5c3 );  UM4( Rc7c3 ); UM4( Rc5c6 ); UM4( Rc6c5 );
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.33)$$

```
> # Using formulas from the artivcle create 6*6
  matrices and compare them with matrices received
  after resolving relations.`
```

```
> SSSS := {};
```

$$SSSS := \varnothing \quad (5.34)$$

```
> #UM(c[1]);UM(c[2]);
> #UM4(c[3]);#
```

```
> i := 'i'; j := 'j'; k := 'k';s := 's';t := 't';
```

$$i := i$$
$$j := j$$
$$k := k$$
$$s := s$$
$$t := t \quad (5.35)$$

```
> H01 := (i,j) -> ( a[1] + 2*i    -j );
```

$$H01 := (i, j) \mapsto a_1 + 2 \cdot i - j \quad (5.36)$$

```
> H10 := (i,j) -> ( a[2] - 2*i + 2*j );
```

$$H10 := (i, j) \mapsto a_2 - 2 \cdot i + 2 \cdot j \quad (5.37)$$

```
> H2  := (i,j) -> H01(i,j) + H10(i,j); # ( j +h[1] + h
  [10])
```

$$H2 := (i, j) \mapsto H01(i, j) + H10(i, j) \quad (5.38)$$

```
> s   := (j,k) -> (a[3] -j + 2*k - 1 );
```

(5.39)

$$s := (j, k) \mapsto a_3 - j + 2 \cdot k - 1 \quad (5.39)$$

```
> #t   := (j,k) -> (a[4] -j + 2*k - 1 );
> Sp  := (i,j,k) -> ( a[1] + a[3] + 2*i  - 2*j + 2*k -
  1  )/2;
```

$$Sp := (i, j, k) \mapsto \frac{a_1}{2} + \frac{a_3}{2} + i - j + k - \frac{1}{2} \quad (5.40)$$

```
> Sm  := (i,k)   -> (-a[1] + a[3] - 2*i        + 2*k -
  1  )/2;
```

$$Sm := (i, k) \mapsto -\frac{a_1}{2} + \frac{a_3}{2} - i + k - \frac{1}{2} \quad (5.41)$$

```
> Tp  := (k)     -> ( a[1] + a[2] + a[4]       + 2*k -
  1  )/2;
```

$$Tp := k \mapsto \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_4}{2} + k - \frac{1}{2} \quad (5.42)$$

```
> Tm  := (j,k)   -> (-a[1] - a[2] + a[4] - 2*j + 2*k -
  1  )/2;
```

$$Tm := (j, k) \mapsto -\frac{a_1}{2} - \frac{a_2}{2} + \frac{a_4}{2} - j + k - \frac{1}{2} \quad (5.43)$$

```
> Qp  := (j,k)   -> nu / s(j,k) + 1;
```

$$Qp := (j, k) \mapsto \frac{\nu}{s(j, k)} + 1 \quad (5.44)$$

```
> Qm  := (j,k)   -> nu / s(j,k) - 1;
```

$$Qm := (j, k) \mapsto \frac{\nu}{s(j, k)} - 1 \quad (5.45)$$

```
> nu;
```

$$0 \quad (5.46)$$

```
> ME01 := (i,j,k) -> Matrix( 6,6,[
> [          Sp(i,j,k  ),0,0,0,0,0],
> [0,        Sp(i,j,k+1),0,0,0,0],
> [0,0,      Sp(i,j,k+2),0,0,0],
> [0,0,0,    Sp(i,j,k+3),0,0],
> [0,0,0,0,  Sp(i,j,k+4),0],
> [0,0,0,0,0,Sp(i,j,k+5)]]);
```

$$ME01 := (i, j, k) \mapsto Matrix(6, 6, [[Sp(i, j, k), 0, 0, 0, 0, 0], [0, Sp(i, j, k+1), 0, 0, 0, 0], [0, 0, Sp(i, j, k+2), 0, 0, 0], [0, 0, 0, Sp(i, j, k+3), 0, 0], [0, 0, 0, 0, Sp(i, j, k+4), 0], [0, 0, 0, 0, 0, Sp(i, j, k+5)]]) \quad (5.47)$$

```
> MF01 := (i,j,k) -> Matrix( 6,6,[
> [          Sm(i,k  ),0,0,0,0,0],
> [0,        Sm(i,k+1),0,0,0,0],
> [0,0,      Sm(i,k+2),0,0,0],
> [0,0,0,    Sm(i,k+3),0,0],
> [0,0,0,0,  Sm(i,k+4),0],
> [0,0,0,0,0,Sm(i,k+5)]]);
```

$$MF01 := (i, j, k) \mapsto Matrix(6, 6, [[Sm(i, k), 0, 0, 0, 0, 0], [0, Sm(i, k+1), 0, 0, 0, 0], [0, 0, Sm(i, k+2), 0, 0, 0], [0, 0, 0, Sm(i, k+3), 0, 0], [0, 0, 0, 0, Sm(i, k+4), 0], [0, 0, 0, 0, 0, Sm(i, k+5)]]) \quad (5.48)$$

```
> MH1 := UM(ME01(-1,0,k) . MF01(0,0,k) - MF01(1,0,k) . ME01(0,0,k));
```

$$MH1 := \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 \end{bmatrix} \quad (5.49)$$

```
> UM( c[1] - MF01(1,0,1).ME01(0,0,1));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.50)$$

```
> ME21 := (i,j,k) -> Matrix( 6,6,[
> [          2*Tm(j+1,k  ),0,0,0,0,0],
> [0,        2*Tm(j+1,k+1),0,0,0,0],
> [0,0,      2*Tm(j+1,k+2),0,0,0],
> [0,0,0,    2*Tm(j+1,k+3),0,0],
```

```
> [0,0,0,0,  2*Tm(j+1,k+4),0],
> [0,0,0,0,0,2*Tm(j+1,k+5)]]);
```

$$ME21 := (i, j, k) \mapsto Matrix(6, 6, [[2 \cdot Tm(j+1, k), 0, 0, 0, 0, 0], [0, 2 \cdot Tm(j+1, k+1), 0, 0, 0, 0], [0, 0, 2 \cdot Tm(j+1, k+2), 0, 0, 0], [0, 0, 0, 2 \cdot Tm(j+1, k+3), 0, 0], [0, 0, 0, 0, 2 \cdot Tm(j+1, k+4), 0], [0, 0, 0, 0, 0, 2 \cdot Tm(j+1, k+5)]]) \quad (5.51)$$

```
> MF21 := (i,j,k) -> Matrix( 6,6,[
> [          2*Tp(k-1),0,0,0,0,0],
> [0,        2*Tp(k  ),0,0,0,0],
> [0,0,      2*Tp(k+1),0,0,0],
> [0,0,0,    2*Tp(k+2),0,0],
> [0,0,0,0,  2*Tp(k+3),0],
> [0,0,0,0,0,2*Tp(k+4)]]);
```

$$MF21 := (i, j, k) \mapsto Matrix(6, 6, [[2 \cdot Tp(k-1), 0, 0, 0, 0, 0], [0, 2 \cdot Tp(k), 0, 0, 0, 0], [0, 0, 2 \cdot Tp(k+1), 0, 0, 0], [0, 0, 0, 2 \cdot Tp(k+2), 0, 0], [0, 0, 0, 0, 2 \cdot Tp(k+3), 0], [0, 0, 0, 0, 0, 2 \cdot Tp(k+4)]]) \quad (5.52)$$

```
> UM(c[2]);
```

$$\begin{bmatrix} -(a_4+1+a_2+a_1)\ (-a_4+1+a_2+a_1) & & \cdots \\ 0 & -(a_4+ & \cdots \\ 0 & & \cdots \\ 0 & & \cdots \\ 0 & & \cdots \\ 0 & & \cdots \end{bmatrix} \quad (5.53)$$

```
> UM(MF21(1,2,1+1) . ME21(0,0,1) - c[2]);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.54)$$

```
> MH21 := UM(ME21(-1,-2,k-1) . MF21(0,0,k) - MF21(1,2,
  k+1) . ME21(0,0,k));
```

MH21 := **(5.55)**

$$\begin{bmatrix} 4\ a_1 + 4\ a_2 & 0 & 0 & 0 & \cdots \\ 0 & 4\ a_1 + 4\ a_2 & 0 & 0 & \cdots \\ 0 & 0 & 4\ a_1 + 4\ a_2 & 0 & \cdots \\ 0 & 0 & 0 & 4\ a_1 + 4\ a_2 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \end{bmatrix}$$

```
> ME10 := (i,j,k) -> Matrix( 6,6,[
> [                              Tm(j+1,k   )*Qm(j+1,k
   ), 0,0,0,0,0 ],
> [          Sm(i,k   )*Qp(j+1,k   ), Tm(j+1,k+1)*Qm(j+1,
  k+1), 0,0,0,0 ],
> [0,        Sm(i,k+1)*Qp(j+1,k+1), Tm(j+1,k+2)*Qm(j+1,
  k+2), 0,0,0 ],
> [0,0,      Sm(i,k+2)*Qp(j+1,k+2), Tm(j+1,k+3)*Qm(j+1,
  k+3), 0,0 ],
> [0,0,0,    Sm(i,k+3)*Qp(j+1,k+3), Tm(j+1,k+4)*Qm(j+1,
  k+4), 0 ],
> [0,0,0,0,  Sm(i,k+4)*Qp(j+1,k+4), Tm(j+1,k+5)*Qm(j+1,
  k+5) ]]);
```

$ME10 := (i, j, k) \mapsto Matrix(6, 6, [[Tm(j+1, k) \cdot Qm(j+1, k), 0, 0, 0, 0, 0], [Sm(i, k) \cdot Qp(j+1, k), Tm(j+1, k+1) \cdot Qm(j+1, k+1), 0, 0, 0, 0], [0, Sm(i, k+1) \cdot Qp(j+1, k+1), Tm(j+1, k+2) \cdot Qm(j+1, k+2), 0, 0, 0], [0, 0, Sm(i, k+2) \cdot Qp(j+1, k+2), Tm(j+1, k+3) \cdot Qm(j+1, k+3), 0, 0], [0, 0, 0, Sm(i, k+3) \cdot Qp(j+1, k+3), Tm(j+1, k+4) \cdot Qm(j+1, k+4), 0], [0, 0, 0, 0, Sm(i, k+4) \cdot Qp(j+1, k+4), Tm(j+1, k+5) \cdot Qm(j+1, k+5)]])$ **(5.56)**

```
> MF10 := (i,j,k) -> Matrix( 6,6,[
> [           Sp(i,j,k   )*Qp(j+1,k   ), Tp(k   )*Qm(j+1,
  k+1),0,0,0,0],
> [0,         Sp(i,j,k+1)*Qp(j+1,k+1), Tp(k+1)*Qm(j+1,
  k+2),0,0,0],
```

```
> [0,0,      Sp(i,j,k+2)*Qp(j+1,k+2), Tp(k+2)*Qm(j+1,
  k+3),0,0],
> [0,0,0,    Sp(i,j,k+3)*Qp(j+1,k+3), Tp(k+3)*Qm(j+1,
  k+4),0],
> [0,0,0,0,  Sp(i,j,k+4)*Qp(j+1,k+4), Tp(k+4)*Qm(j+1,
  k+5)],
> [0,0,0,0,0,Sp(i,j,k+5)*Qp(j+1,k+5)  ]]);
```

$MF10 := (i, j, k) \mapsto Matrix(6, 6, [[Sp(i, j, k) \cdot Qp(j+1, k), Tp(k) \cdot Qm(j+1, k+1), 0, 0, 0, 0], [0, Sp(i, j, k+1) \cdot Qp(j+1, k+1), Tp(k+1) \cdot Qm(j+1, k+2), 0, 0, 0], [0, 0, Sp(i, j, k+2) \cdot Qp(j+1, k+2), Tp(k+2) \cdot Qm(j+1, k+3), 0, 0], [0, 0, 0, Sp(i, j, k+3) \cdot Qp(j+1, k+3), Tp(k+3) \cdot Qm(j+1, k+4), 0], [0, 0, 0, 0, Sp(i, j, k+4) \cdot Qp(j+1, k+4), Tp(k+4) \cdot Qm(j+1, k+5)], [0, 0, 0, 0, 0, Sp(i, j, k+5) \cdot Qp(j+1, k+5)]])$ **(5.57)**

```
> for ii from 1 to 5 do e[ii,ii+1] := UU( Tp(ii) * Tm(0,
  ii) * Qm(0,ii) * Qm(1,ii+1)); od;
```

$$e_{1,2} := -\frac{(a_4+1+a_2+a_1)\ (-a_4-1+a_2+a_1)}{4}$$

$$e_{2,3} := -\frac{(a_4+a_1+a_2+3)\ (-a_4+a_1+a_2-3)}{4}$$

$$e_{3,4} := -\frac{(a_4+5+a_2+a_1)\ (-a_4-5+a_2+a_1)}{4}$$

$$e_{4,5} := -\frac{(a_4+7+a_2+a_1)\ (-a_4-7+a_2+a_1)}{4}$$

$$e_{5,6} := -\frac{(a_4+9+a_2+a_1)\ (-a_4-9+a_2+a_1)}{4}$$ **(5.58)**

```
> MH10 := UM4(ME10(0,-1,k) . MF10(0,0,k) - MF10(0,1,k)
  . ME10(0,0,k));
```

$$MH10 := \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & a_2 \end{bmatrix}$$ **(5.59)**

```
> UM4( c[3] - MF10(0,1,1).ME10(0,0,1));
```

(5.60)

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.60}$$